Basics of Electron Storage Rings

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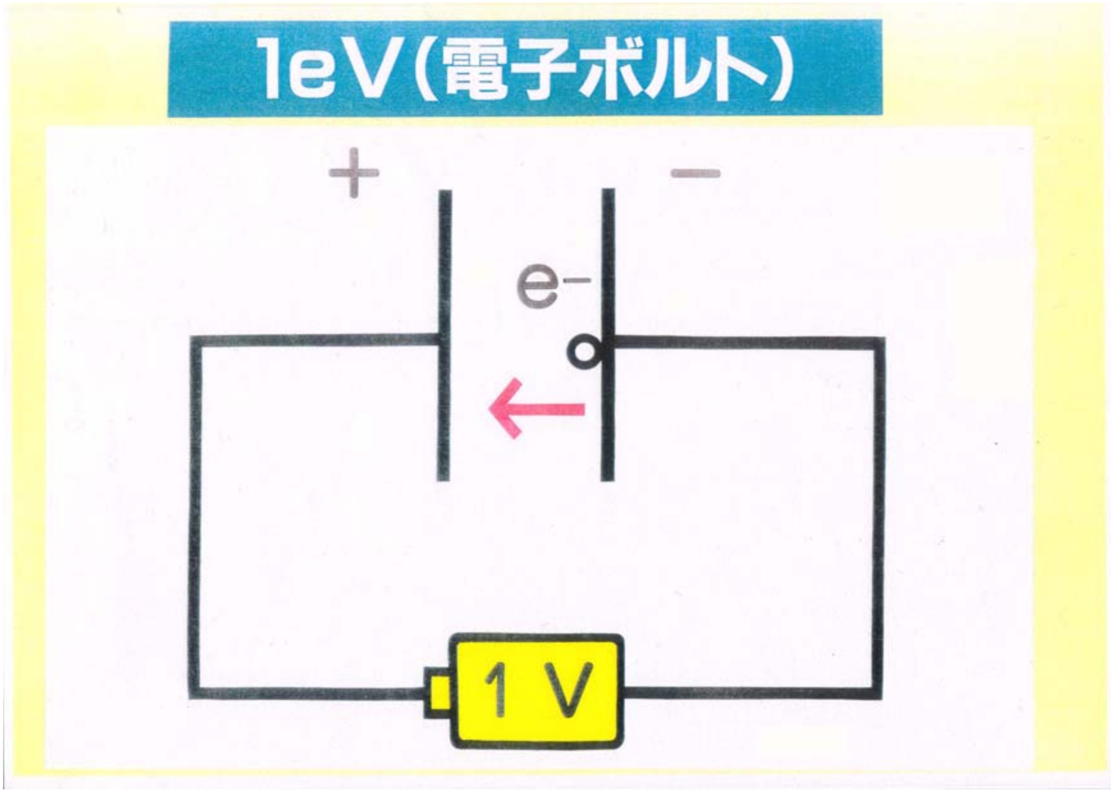
1. What is an accelerator ?

- Accelerator is a machine that gives energy to charged particles.

- The simplest way to accelerate charged particles is to use electrostatic voltage.

- In order to get 2.0 GeV, the energy of SESAME, we need to apply 2.0 GV between a gap; this is not at all possible.

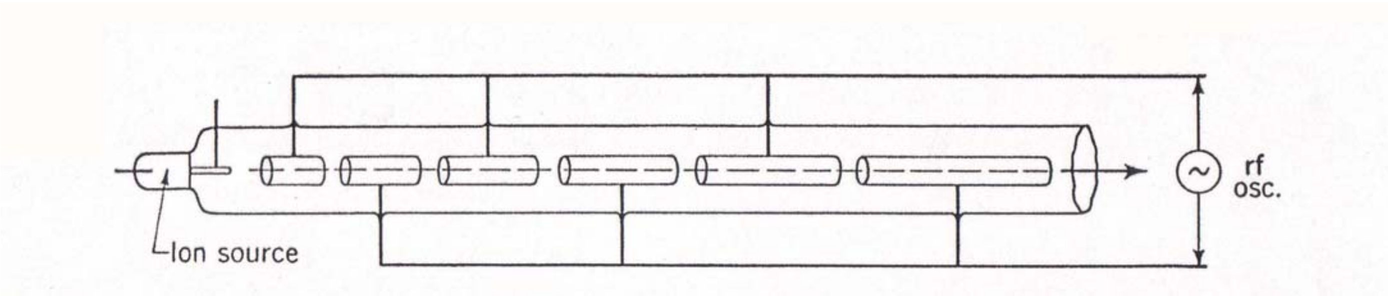


The simplest way to accelerate charged particle is to use electrostatic voltage

2. Alternating current accelerator

- Wideroe invented an alternating current accelerator in 1928 and paved the way to higher energy accelerators.

- On the basis of Wideroe’s idea cyclotron was invented by E.O. Lawrence in 1932, and then synchrotron by E.M. McMillan in 1945.

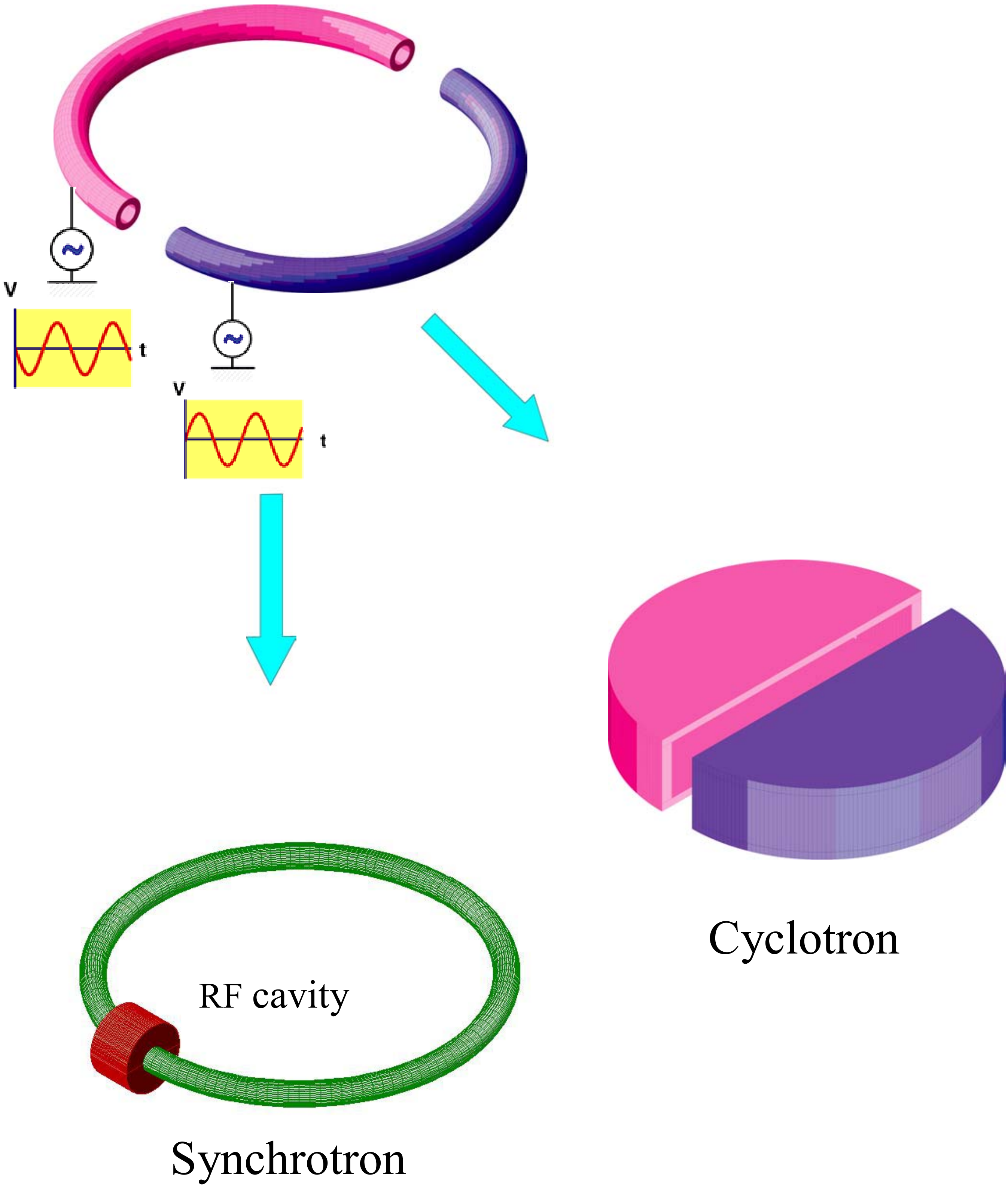


Early linear accelerator for heavy ions with accelerating electrode lengths increasing with the square roots of a series of integers.

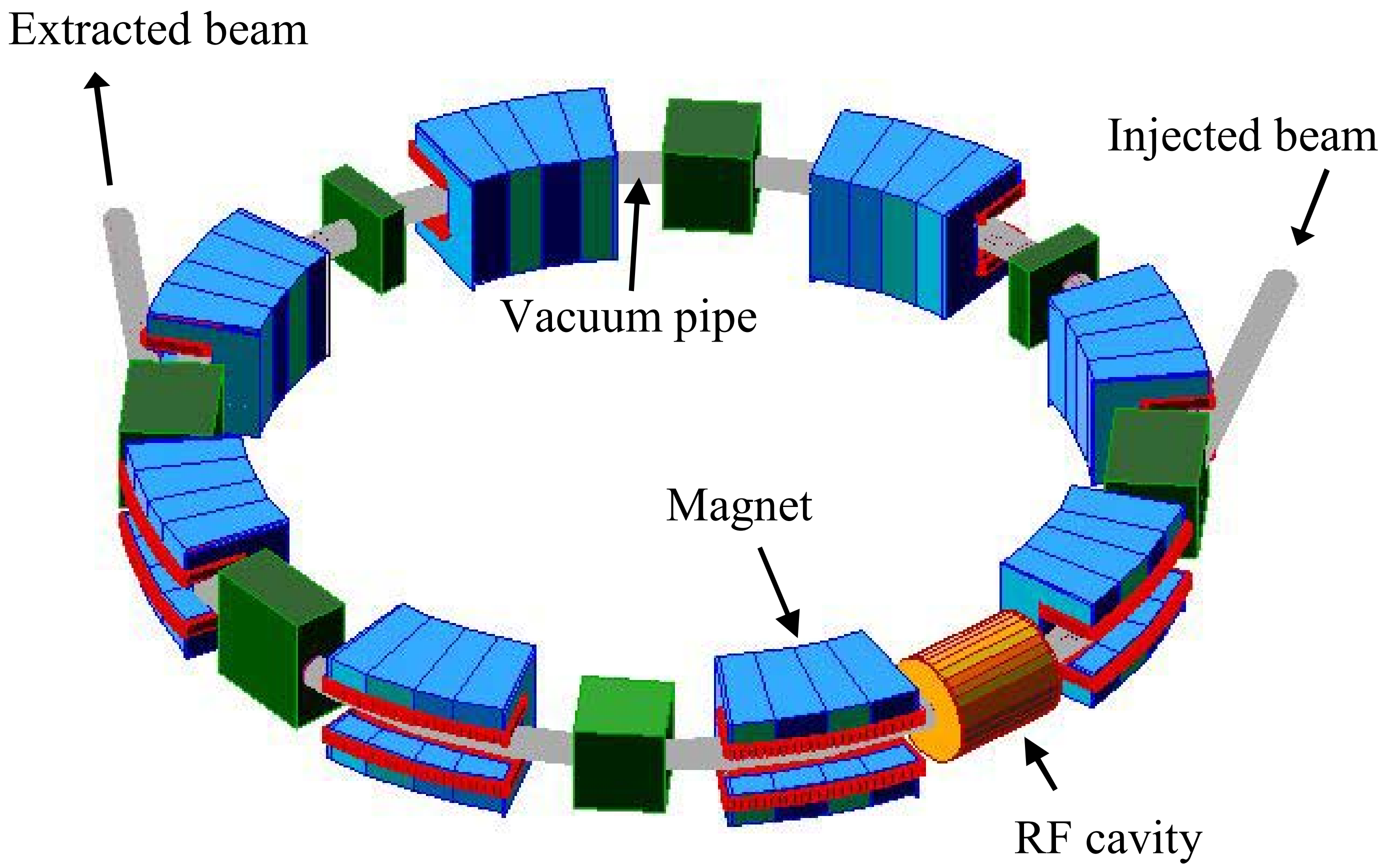
3. Synchrotron and storage ring

- Figure shows the basic idea of synchrotron: it consists of vacuum pipes, magnets, and radio frequency (rf) accelerating cavities. Beam is injected into the synchrotron and extracted from it.

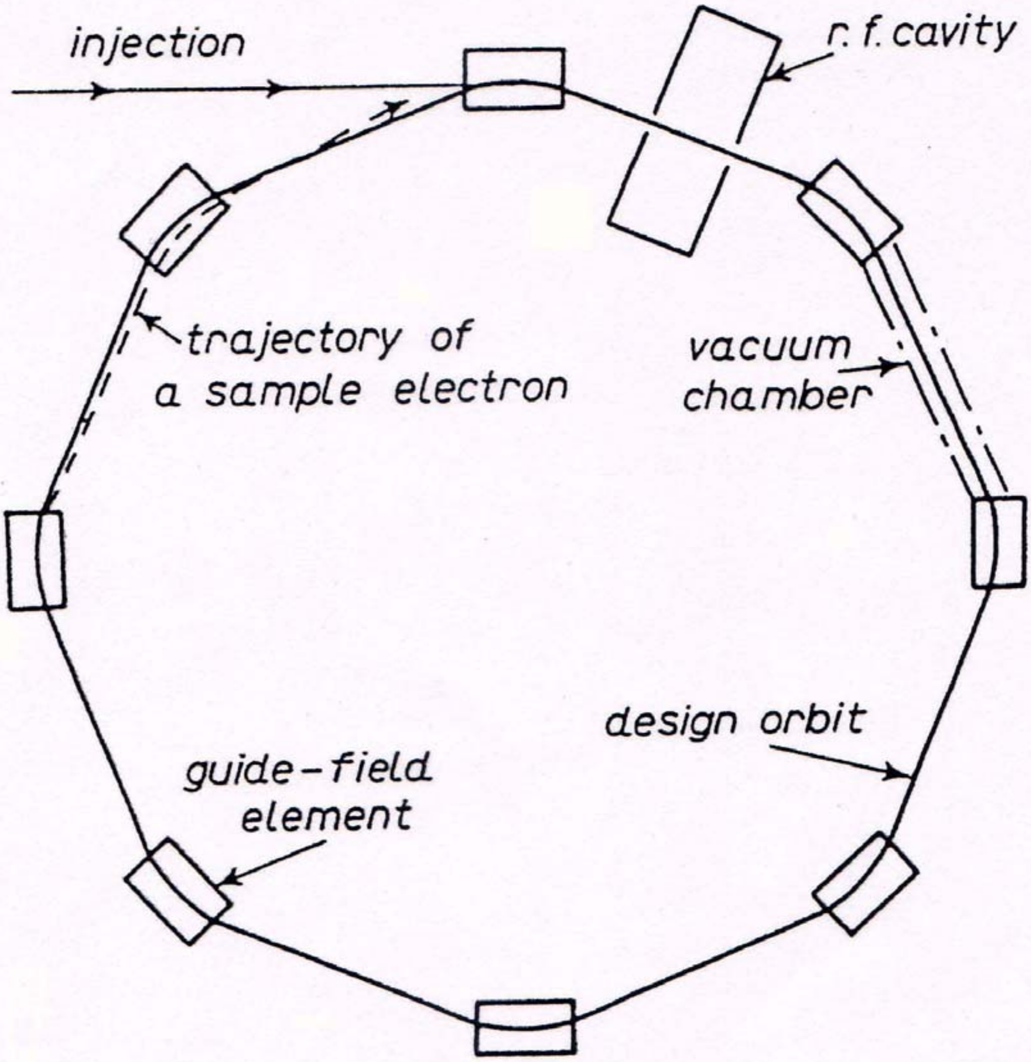
- Storage ring is a kind of synchrotron, where injected beam is stored and not extracted.



From Wideroe linac to cyclotron and synchrotron



Synchrotron



Schematic diagram of an electron storage ring.

4. Magnets

- Magnets are most fundamental components of the storage ring. Usually magnets are electro-magnets.

- A few kinds of magnets are used in the storage ring: dipole magnet, quadrupole magnet, sextupole, magnet, and octupole magnet.

電子リング（外側）と陽電子リング（内側）

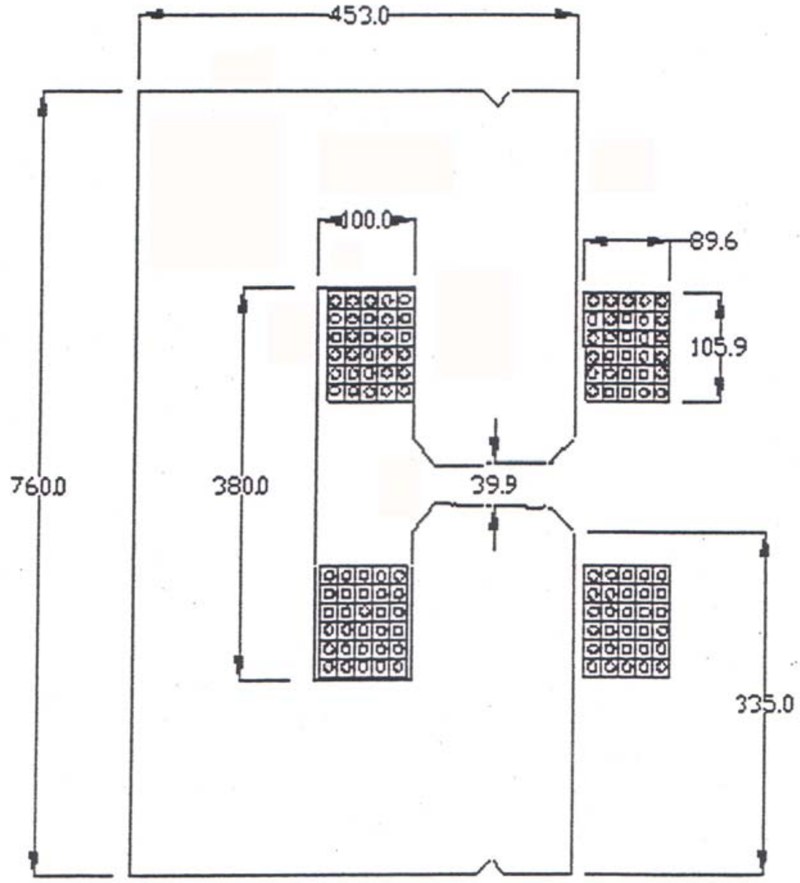
Two storage rings of KEKB

- The function of the dipole magnet is to bend the particle. Basic formula of the bending is:

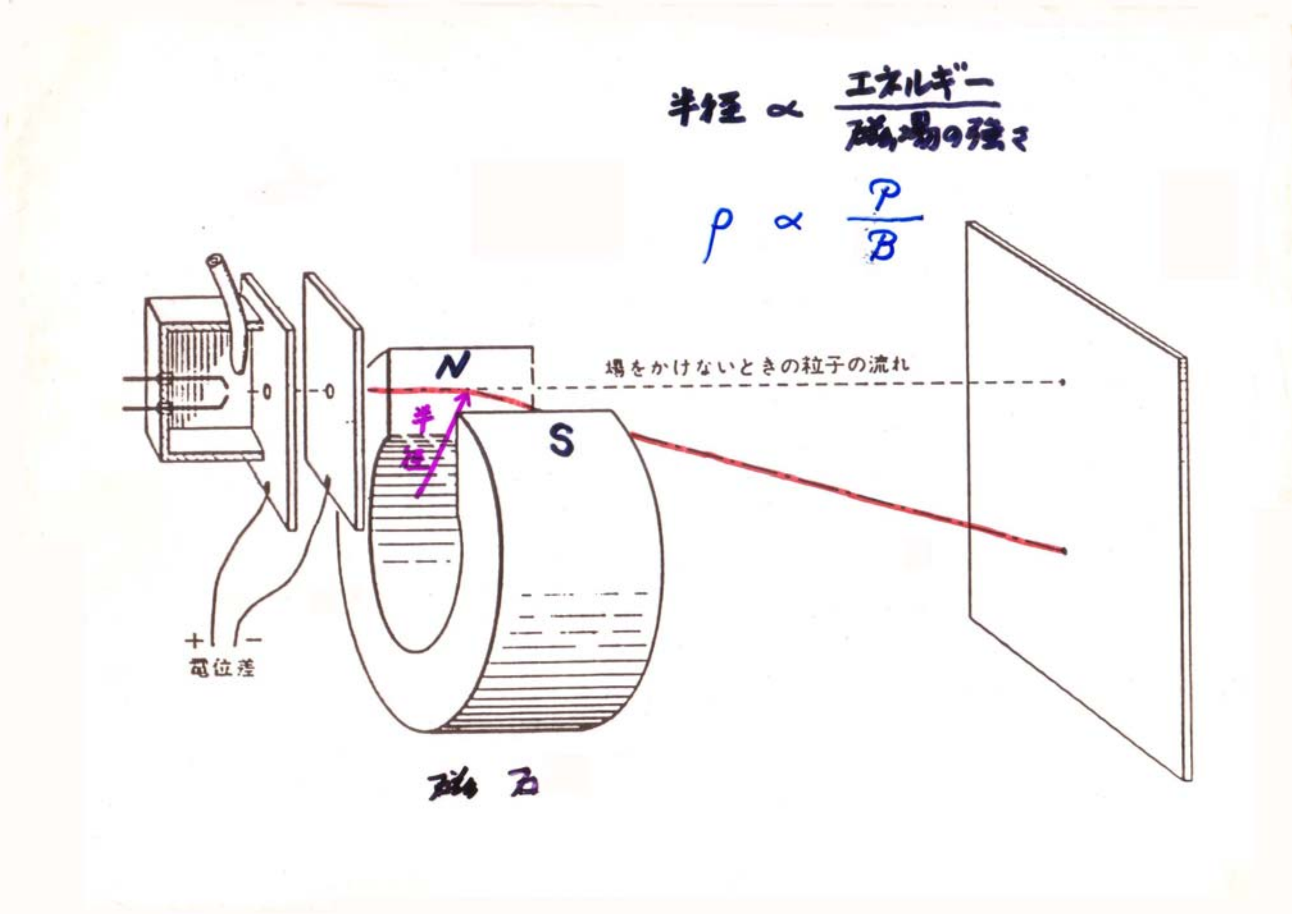
$$\frac{1}{\rho}\left[m^{-1}\right] = 0.2998\frac{B\left[Tesla\right]}{\beta E\left[GeV\right]}$$

- where ρ is the bending radius, B the magnetic field, E the energy of the particle, and β is 1/c (electron case β=1).

- For SESAME case, B=1.35 Tesla, E=2.0 GeV, and ρ becomes 4.9 m.



Cross section of dipole magnet



Function of dipole field

- Quadrupole magnet works as a lens. Principle of the lens is shown in the Figure.

- Different from an optical lens, the quadrupole magnet cannot focus the beam in both horizontal and vertical directions simultaneously. If it focuses the beam in the horizontal direction, it works as a defocusing lens in the vertical direction.

- We can define a focusing strength *k* by

$$k\left[m^{-2}\right] = 0.2998\,\frac{g\left[Tesla/m\right]}{\beta E\left[GeV\right]}$$

where, *g* is the field gradient of the quadrupole magnet.

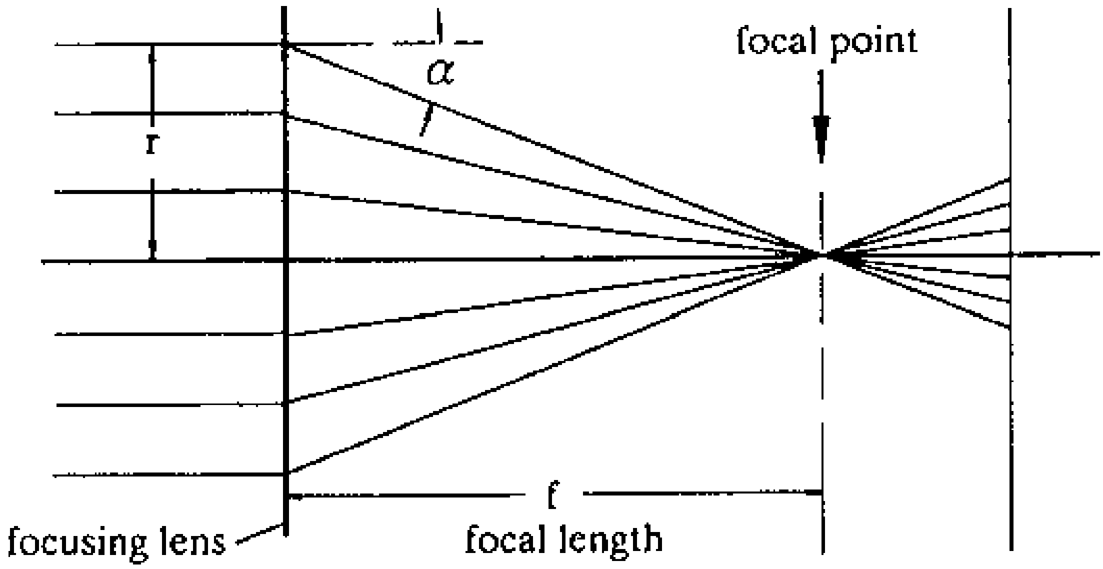
- The focal length of the quadrupole is given by *k* as:

$$\frac{1}{f} = kl$$

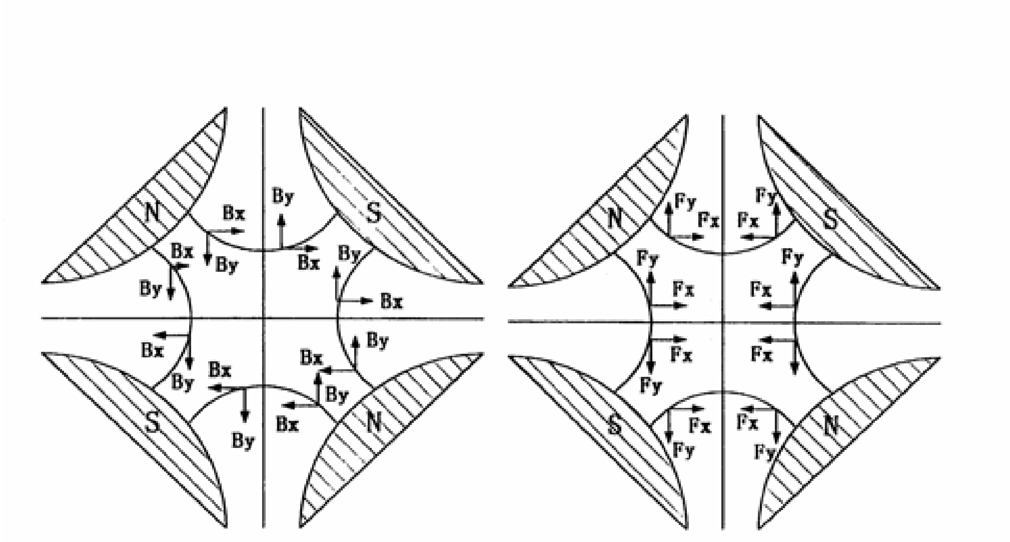
where l is the path length of the particle in the magnet.

- Typical value of g is 10 Tesla/m: at 2 GeV and l of 0.5 m, f becomes 1.33 m.

- In order to focus the beam in both directions simultaneously, we need to have at least two quadrupoles.



Action of a lens

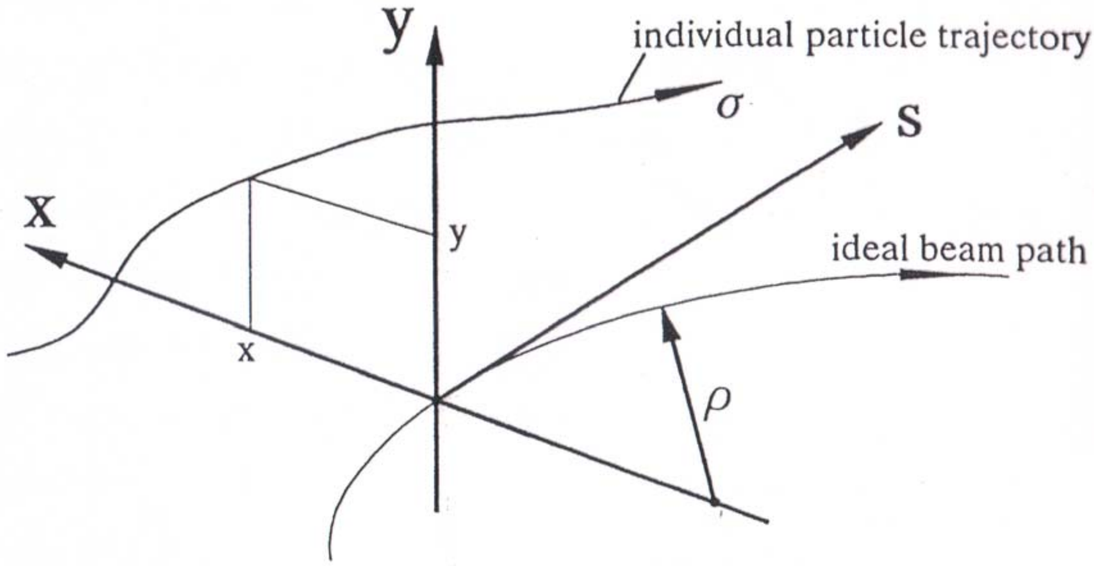


Components and force in a quadrupole.
(Negatively charged particles enter the plane
of the paper with paths perpendicular.)

5. Equation of motion

- We need to use a special coordinate system.

- Equation of motion is usually a differential equation with respect to time t, however, we are usually interested not in time but particle trajectory along a path. We, therefore, write down equations of motion with respect to distance s ($=\beta t$).



Coordinate system

- The equation of motion in the approximation of linear beam dynamics is:

$$x'' + \left(\frac{1}{\rho^2} + k \right) x = 0$$

$$y'' - ky = 0$$

- The term $1/\rho^2$ comes from the fact that dipole magnet field also has a focusing action in the horizontal direction.

- This equation can be generalized as:

$$u'' + Ku = 0$$

If K is constant, this is a harmonic oscillator, and principal solutions are expressed in matrix formulation:

$$\begin{bmatrix} u(s) \\ u'(s) \end{bmatrix} = \begin{bmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$

- In a drift space where there is no magnet this matrix is expressed by

$$\begin{bmatrix} u(s) \\ u'(s) \end{bmatrix} = \begin{bmatrix} 1 & s - s_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$

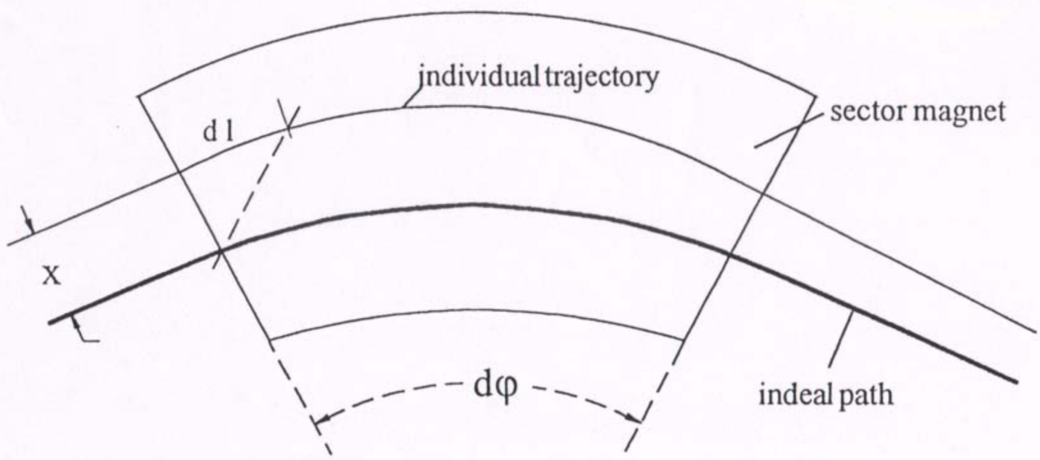


Fig. 4.16. Path length in a sector magnet

- In a quadrupole magnet the matrix for focusing case is:

$$\begin{bmatrix} u(s) \\ u'(s) \end{bmatrix} = \begin{bmatrix} \cos\psi & \dfrac{1}{\sqrt{k_0}}\sin\psi \\ -\sqrt{k_0}\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$

where

$$\psi = \sqrt{k_0}\left(s - s_0\right)$$

- In the plane of defocusing case, we get

$$\begin{bmatrix} u(s) \\ u'(s) \end{bmatrix} = \begin{bmatrix} \cosh\psi & \dfrac{1}{\sqrt{|k_0|}}\sinh\psi \\ \sqrt{|k_0|}\sinh\psi & \cosh\psi \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$

- If the length of the magnet l is much smaller than the focal length, we may use the so called thin-lens approximation. In this approximation, we set

$$l \to 0 \ ,$$

while keeping the focal strength f constant,

$$f^{-1} = k_0 l = const.$$

as a consequence,

$$\varphi = \sqrt{k_0} l \to 0$$

and matrices can be written as,

$$\begin{bmatrix} u \\ u' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$, focusing

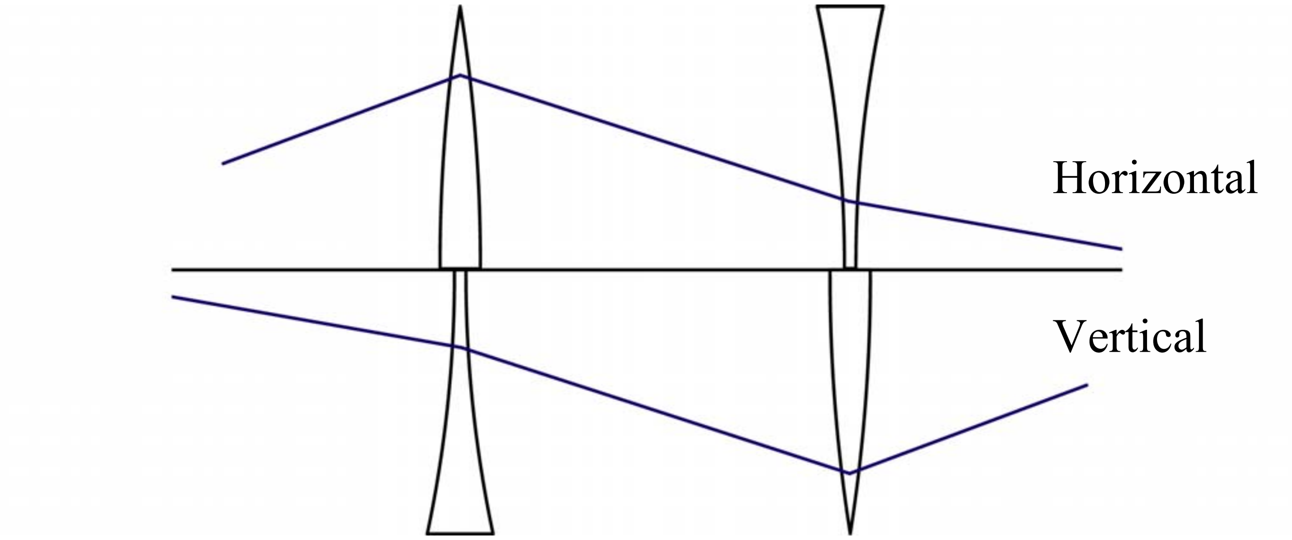
$$\begin{bmatrix} u \\ u' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}$$, defocusing.

- Quadrupole doublet composed of two quadrupole magnets separated by a drift space of length L. In the thin lens approximation, if we assume that the focal length of the quadrupoles are the same, matrix becomes,

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - L/f & L \\ -L/f^2 & 1 - L/f \end{bmatrix}$$

You can find that this doublet is focusing in both directions (see – sign at [2,2] elememt).



Principle of focusing by a quadrupole doublet

6. Dynamics in periodic closed lattices

- Particle beam dynamics in periodic system is determined by the equation of motion (Hill's equation)

$$u'' + K(s) \cdot u = 0$$

where $K(s)$ is periodic with the period of L_p

$$K(s) = K(s + L_p)$$

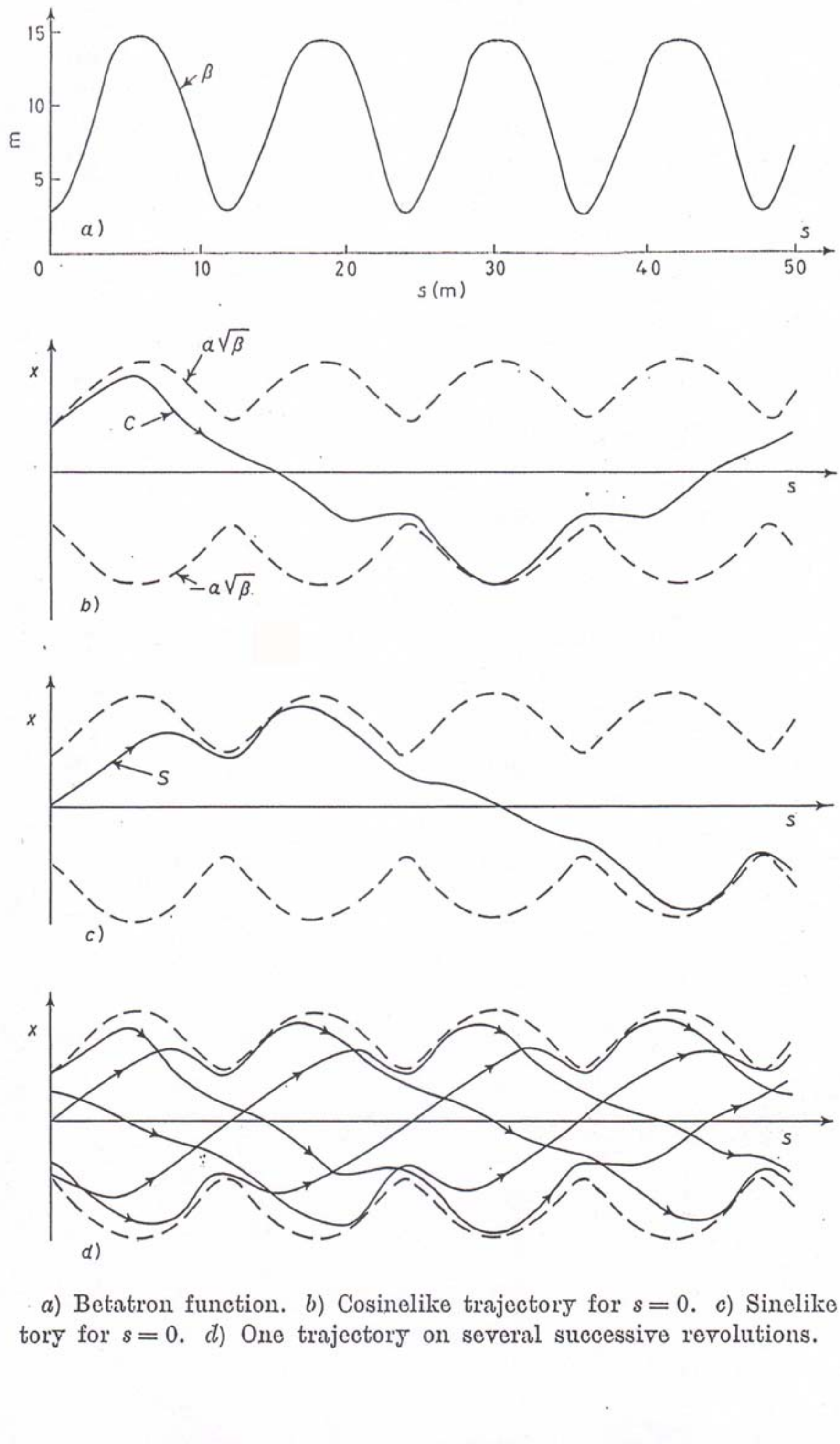
Solution of the Hill's equation can be written by

$$u(s) = a\sqrt{\beta(s)} \cdot e^{\pm i\psi}$$

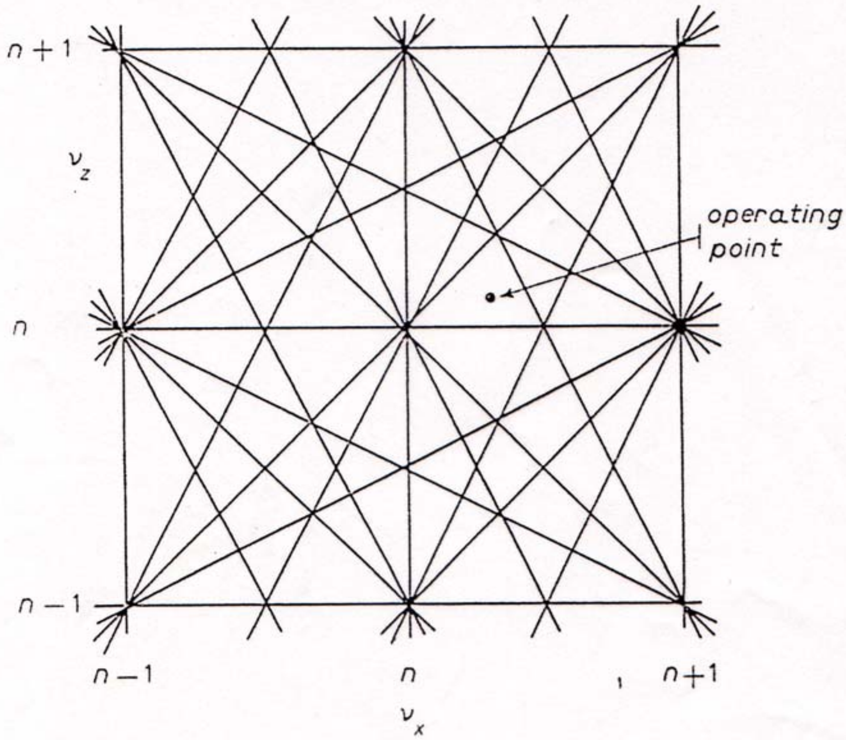
where $\beta(s)$ is periodic function with a period of L_p (Usually L_p is the circumference of the ring, C). ψ is called betatron phase advance and is given by

$$\psi(s - s_0) = \int_{s_0}^{s} \frac{d\tau}{\beta(\tau)}$$

- Phase advance per ring in the unit of 2π is called the betatron tune, ν_x and ν_y . The betatron tunes are the number of transverse oscillations per ring. We should be careful to select good values of these betatron tunes in order to avoid resonances.



a) Betatron function. *b*) Cosinelike trajectory for $s = 0$. *c*) Sinelike tra tory for $s = 0$. *d*) One trajectory on several successive revolutions.



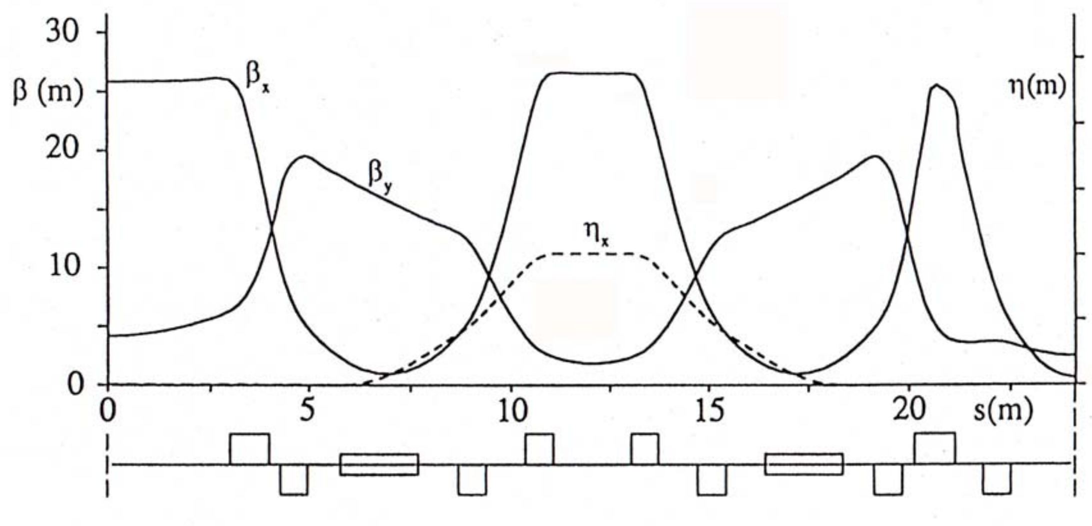
- Lower-order resonance lines on a ν_x, ν_z diagram.

7. Dispersion

- If the particle has a different energy from the central value, the trajectory of the particle differs by,

$$\delta\eta(s)$$

where $\delta = \Delta p/p$ and $\eta(s)$ is called dispersion function. $\eta(s)$ is also a periodic function with a period of C (circumference of the ring).



β_x, β_y, and η_x

8. Emittance

- In the storage ring, particles are circulating in bunch that is an assembly of particles. We, therefore, need to consider the assembly of particles.

- Liouville's theorem states that under the influence of conservative forces the density of the particles in phase space stays constant. This means that the area in the phase space is constant: this area is called the emittance. The emittance is a very important idea for the electron storage ring. The emittance has the dimension of the length, and is usually measured in the unit of 10^{-9} m, which is nano-meter. Modern storage rings for synchrotron light sources have a horizontal emittance of a few nm to a few times 10 nm.

- The phase space ellipse is described by

  $$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon \quad ,$$

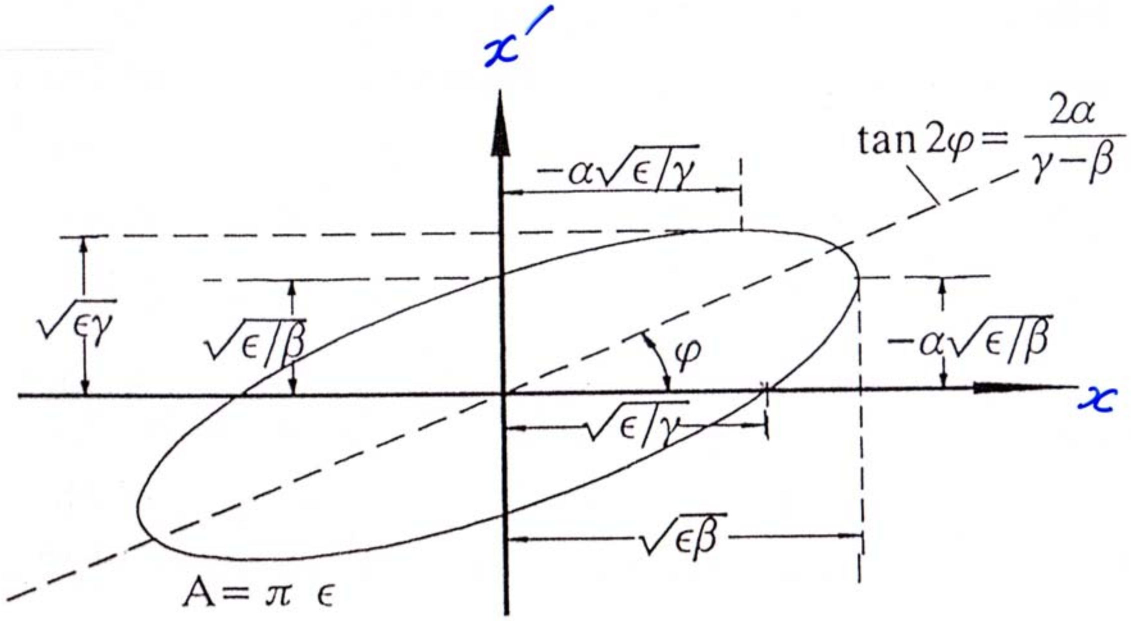
  where ε is the emittance.

- If we knew the emittance we can calculate the beam size in x and y direction σ_x and σ_y from the emittance and β

  $$\sigma_x = \sqrt{\varepsilon_x \beta_x}$$

  $$\sigma_y = \sqrt{\varepsilon_y \beta_y}$$

- The emittance of the electron storage ring is determined by a balance between damping and excitation due to emitting photons. I will cover this topics later.



Phase ellipse

9. Mometum compaction factor

- Within a dipole magnet higher momentum particle has a larger radius of curvature and the lower momentum particle a smaller radius of curvature. This leads to the difference of the path length between different energy particles. The momentum compaction is defined as:

$$\alpha_c = \frac{\Delta L / L_0}{\delta}$$

α_c can be written by using the dispersion function η by

$$\alpha_c = \frac{1}{C}\oint \frac{\eta}{\rho} \cdot ds$$

Usually α_c has a value of 10^{-2} to 10^{-4} (at SESAME, 0.006-0.009). In the electron storage ring, α_c is positive. This means that the higher momentum particle has longer path length.

10. Acceleration and synchrotron oscillation

- Application of radio frequency fields (rf fields) has become exceptionally effective for the acceleration of charged particles. When the particle passes the rf cavity, it gets energy from the rf field.

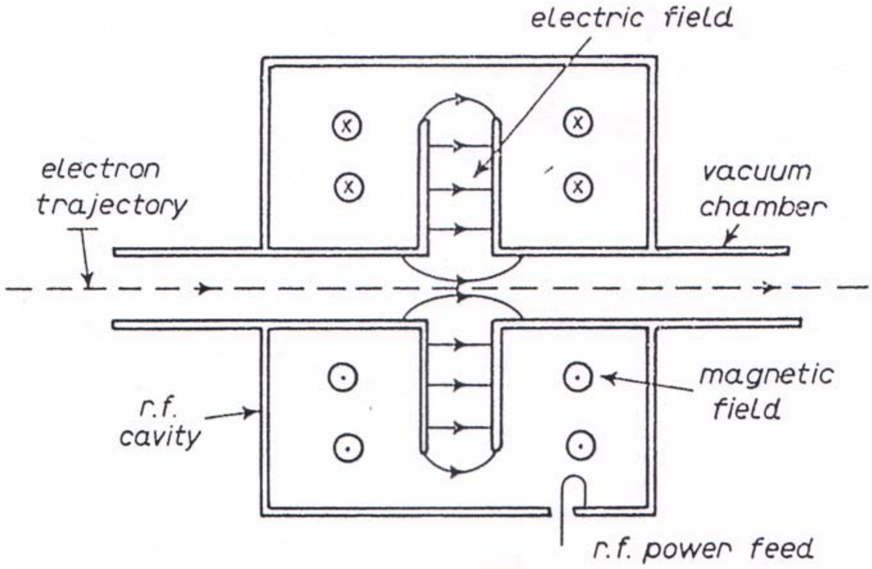
- In the electron storage ring, the energy of the particle is lost by U_0 per turn due to the emission of synchrotron light. The particle should be at some phase (synchronous phase ψ_s) of the rf field in order to get the same energy as that lost by SR.

- Synchrotron oscillation is the oscillation of the energy and phase of the particle. If $\delta > 0$ then it takes longer time to make one turn due to the positive momentum compaction

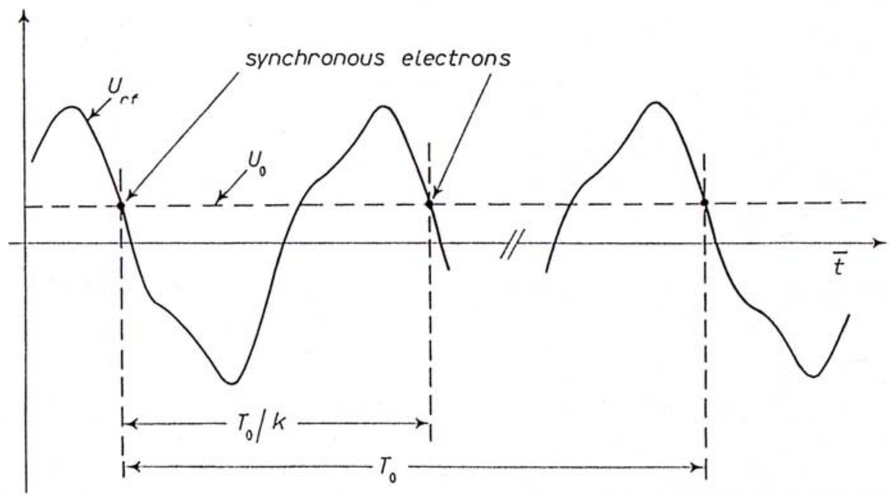
→ $\phi = \psi - \psi_s$ increases → energy decreases → phase increases (see figure). By this mechanism (phase stability) the energy and phase of the particle oscillates. This is called the synchrotron oscillation. The frequency of the synchrotron oscillation Ω is given by,

$$\Omega^2 = \frac{\alpha_c}{T_0} e \frac{V_0}{E_0}$$

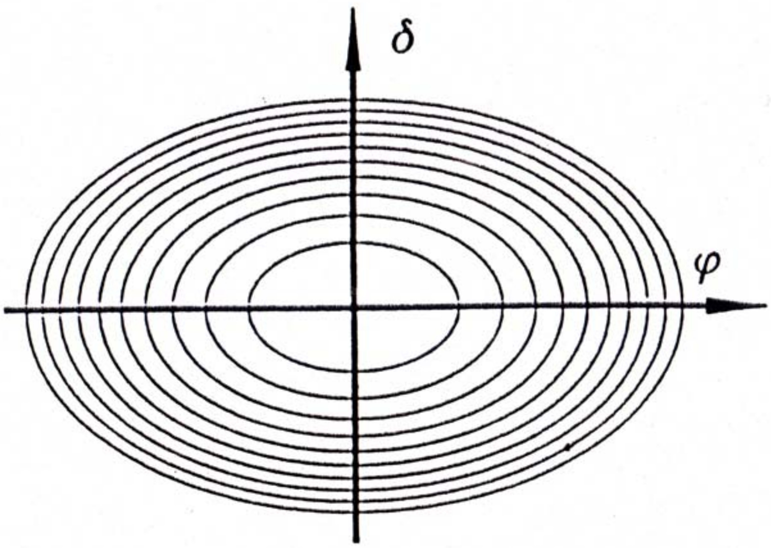
- By this mechanism, in the storage rings, electrons are circulating as bunches (assembly of electrons).



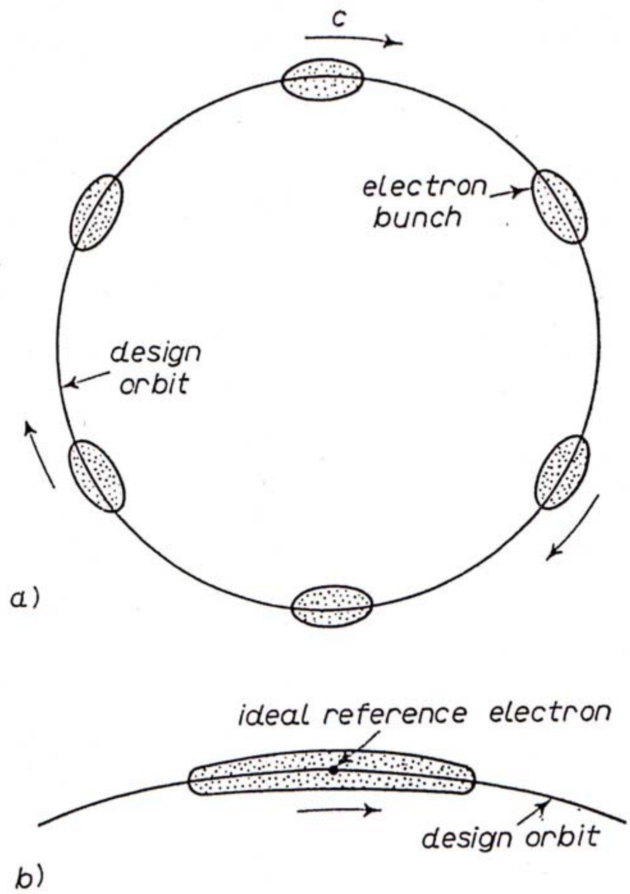
Schematic diagram of a r.f. accelerating cavity.



Acceleration by RF voltage



Synchrotron oscillation of δ, and φ



Circulating bunches in a stored beam.

11. Radiation damping

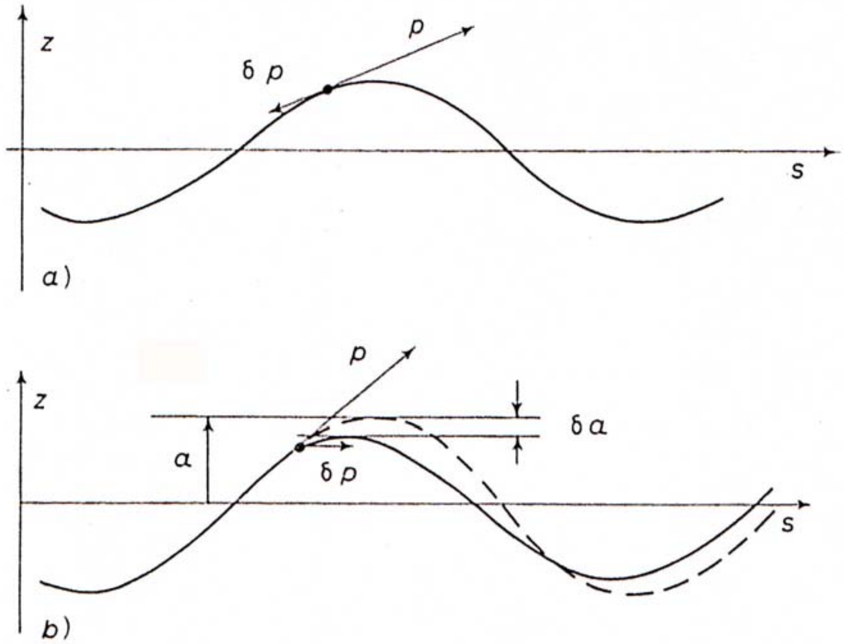
- The characteristics of synchrotron radiation will be covered by Professor Winick's lectures.

- For the moment we assume that SR is emission of photons with the energy u_c (the critical energy), which is given by,

$$u_c(keV) = 0.665 B(Tesla) E^2 (GeV)$$

at B=1.35 Tesla and E=2.0 GeV,

$$u_c = 3.6 keV$$



Effect of an energy change on the vertical betatron oscillations:
tion loss, *b*) for r.f. acceleration.

- The transverse particle oscillation (betatron oscillation) is damped by synchrotron radiation. The reason is that by emitting photon the particle looses its momentum along the moving vector, whereas this energy (momentum) loss is recovered by the accelerating cavity parallel to the beam axis. The net effect is the reduction of the momentum vector in vertical direction.

- Before we assumed that the energy loss due to SR is U_0 and constant. This is not true. The energy loss due to SR is dependent on the energy itself,

$$U_{rad}(\varepsilon) = U_0 + D\varepsilon \quad ,$$

where ε is the energy deviation. This D is positive, since the energy loss due to SR is:

$$U_{rad}(\varepsilon) \propto E_0^4 / \rho$$

- Due to this $D\varepsilon$ term, the energy (and phase) oscillation is also damped by SR. This is called the radiation damping.

12. Radiation excitation

- SR is quantum emission. When a quantum of energy u is emitted, the energy of the electron is suddenly decreased by the amount of u. This impulsive disturbances – occurring at random times – causes the energy oscillation to grow. This growth is limited – on the average – by the damping. This mechanism determines the energy spread σ_ε:

$$\sigma_\varepsilon \approx \sqrt{E_0 u_c}$$

- Bunch length is proportional to the energy spread:

$$\sigma_\tau = \frac{\alpha_c}{\Omega E_0} \sigma_\varepsilon$$

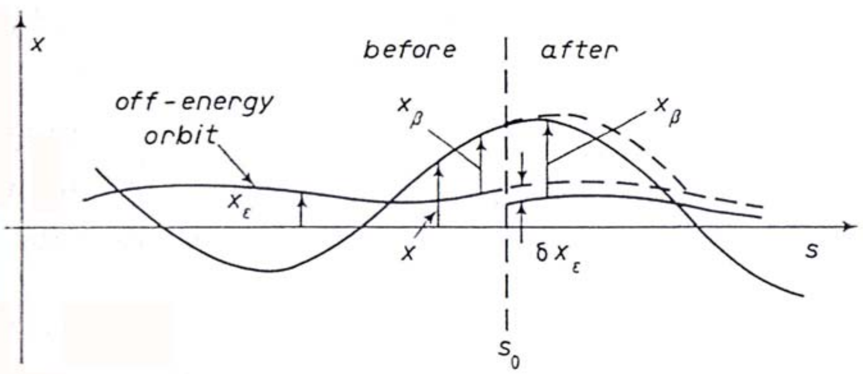
- The emission of discrete quanta in SR will also excite random betatron oscillations, and these quantum-induced oscillations are responsible for the lateral extent of a stored electron beam.

- The emission of a quantum of energy u will result in a change δx_b in the betatron displacement and a change $\delta x'_b$ in the betatron slope given by,

$$\delta x_\beta = -\eta \frac{u}{E_0}$$

$$\delta x'_\beta = -\eta' \frac{u}{E_0}$$

Please note that if h and h' is zero, there is no change in δx_b and $\delta x'_b$. Please consider the case where there is no betatron oscillation (the particle is just on the central trajectory) before emitting photon. After emitting photon, now the particle is off the central trajectory by δx_b and $\delta x'_b$, and you may understand that this excites the betatron oscillation.



Effect of a sudden energy change at s_0 on the betatron displacement.

- The result is given by,

$$\varepsilon_x = \frac{\sigma_x^2}{\beta_x} \propto \gamma^2 \frac{\left\langle 1/\rho^3 \cdot H \right\rangle}{\left\langle 1/\rho^2 \right\rangle},$$

where ε_x is the emittance and H is defined as:

$$H(s) = \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2 \ ,$$

and < > means the average over the ring.

- In the vertical direction, if the machine is perfectly made, there is no dispersion. This means that in an ideal case the beam size in the vertical direction should be zero. In reality, however, due imperfections of the machine such as misalignment, etc., there appears a small residual dispersion in the vertical direction. This causes a finite vertical beam size in the vertical direction. Usually the ration of the vertical emittance to the horizontal emittance is of the order of a few percent.

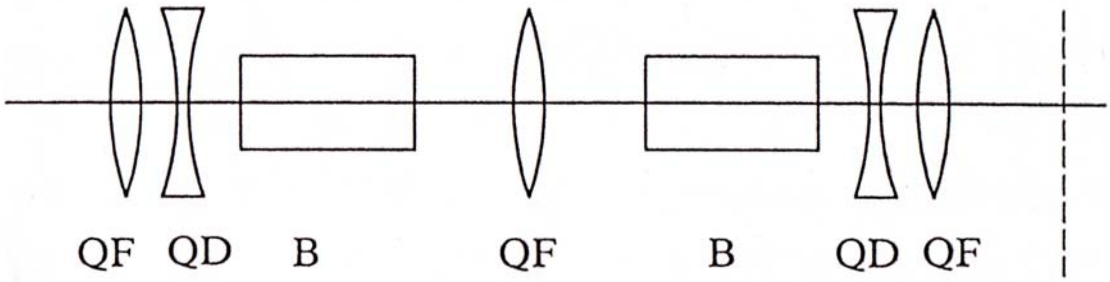
13. Double bend achromat lattice

- The double bend achromat or DBA lattice is designed to make full use of the minimization of beam emittance by the proper choice of lattice function.

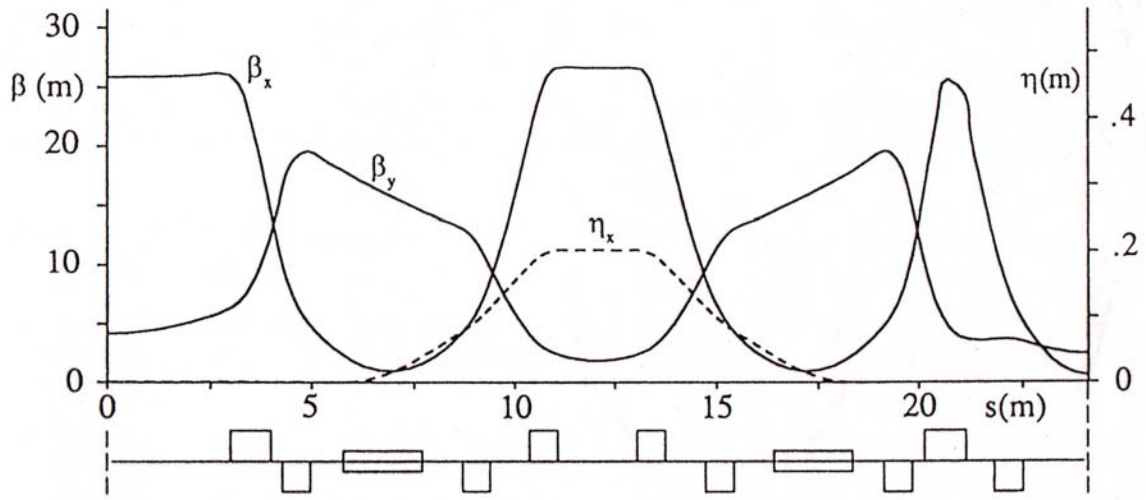
- The ideal beam emittance in this DBA is given,

$$\varepsilon_{DBA}(m) = 5.036 \times 10^{-13} E^2(GeV^2)\Theta^3(\deg^3)$$

If E=2.0 GeV and Θ is 22.5 degree, the emittance becomes 23 nm.



Double bend achromat (DBA) lattice (schematic)



β_x, β_y, and η_x of one cell of DBA lattice