

Collective Instability of a Superbunch - Transverse head-tail instability -

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KEK

Introduction

Air-bag Model

Analysis by the Vlasov equation

Analysis by tracking

Water-bag Model

Analysis by tracking

Summary

Introduction

Collective instabilities

Longitudinal

Single bunch

Transverse

Coupled bunch

Ep instability

What is the impact of superbunch

on circular accelerators?

Ep instability <--- F. Zimmermann

Longitudinal instability <--- Y.H. Chin, H. Tsutsui

Transverse head-tail instability is conceivable.

Examples: CERN-PS, Rutherford-ISIS, KEK-PS, ...

Analysis by the Vlasov equation

Vlasov equation

$$\frac{\partial \Psi}{\partial s} + y' \frac{\partial \Psi}{\partial y} + p_y' \frac{\partial \Psi}{\partial p_y} + z' \frac{\partial \Psi}{\partial z} + \delta' \frac{\partial \Psi}{\partial \delta} = 0$$

Phase space
distribution density

Ψ

expanded by
oscillation modes

Single particle motion

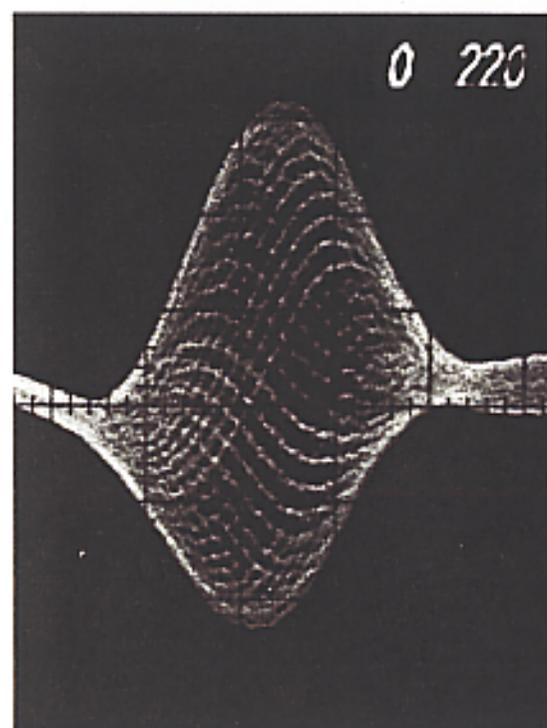
$$y' = p_y$$

$$p_y' = -\left(\frac{\omega\beta}{c}\right)^2 y - \frac{De^2}{cT_0E} e^{-i\Omega s/c} \sum_{p=-\infty}^{\infty} \rho(p\omega_0 + \Omega) Z(p\omega_0 + \Omega) e^{i(p\omega_0 + \Omega)z/c}$$

$$z' = -\eta\delta$$

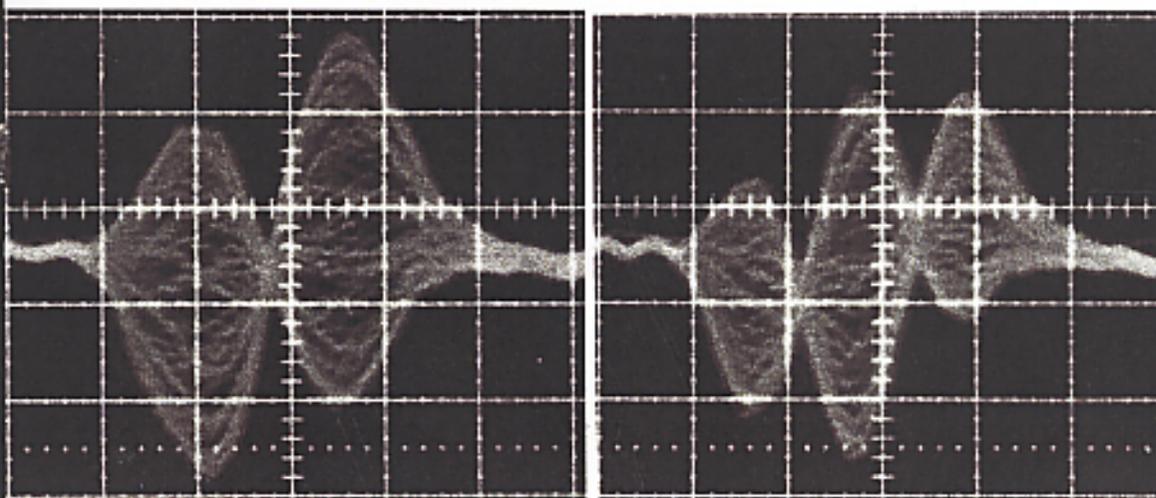
$$\delta' = -\frac{\partial U}{\partial z}$$

Examples: CERN-PS, Rutherford-ISIS, KEK-PS, ...



Mode 0

KEK 12Gev PS
@beginning of acceleration



Mode 1

Mode 2

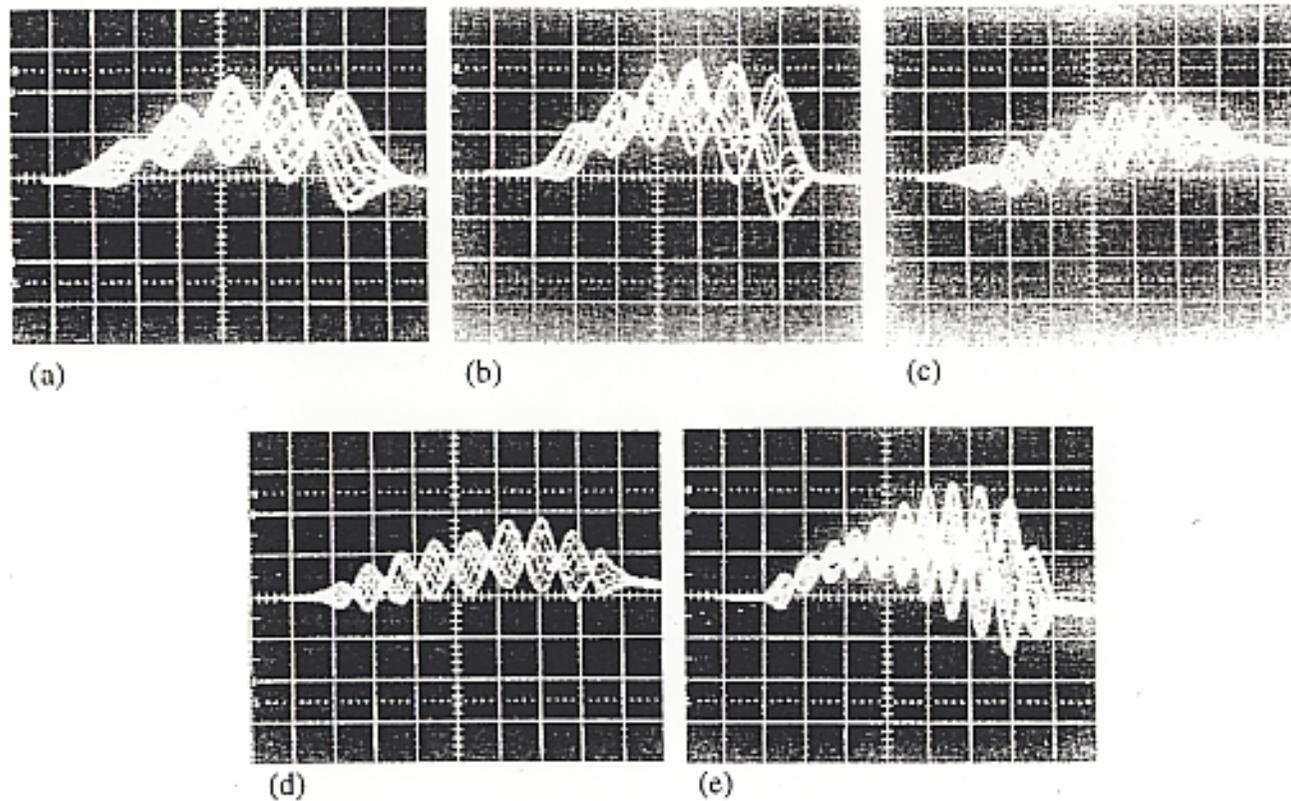
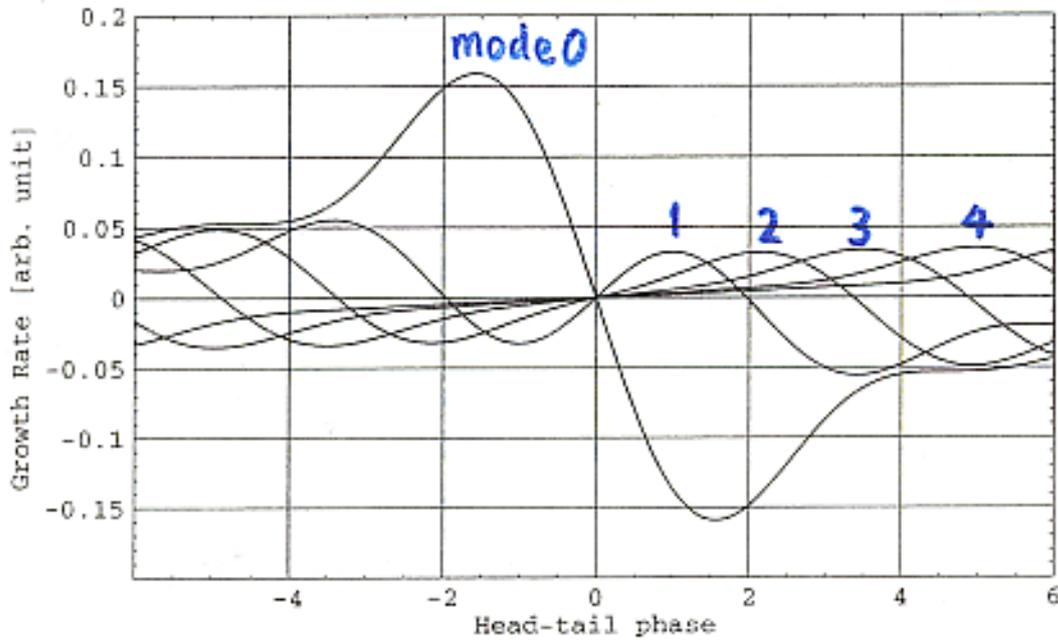


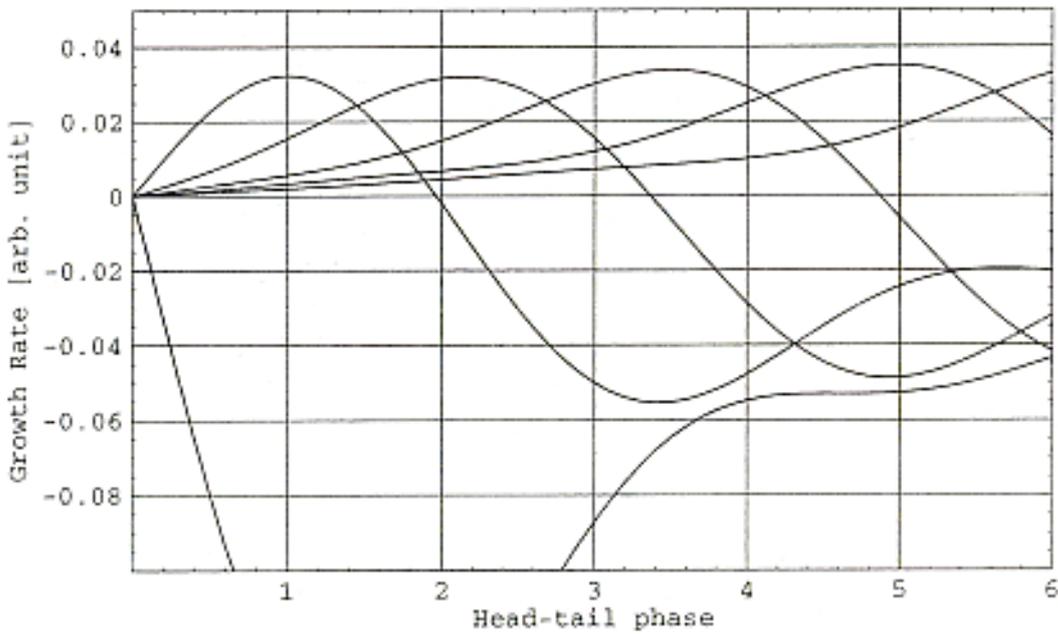
FIGURE 6. ΔR signal from a radial beam-position monitor during 20 consecutive turns. Time scale: 20 ns/div. (a) Nominal single-bunch beam with $Q_x = 6.08$, $Q_y = 6.32$, $\xi_x = -0.5$ and $\xi_y = -1.5$. (b) $Q_x = 6.18$, $Q_y = 6.21$, $\xi_x = -0.7$ and $\xi_y = -1.7$. (c) $Q_x = 6.21$, $Q_y = 6.18$, $\xi_x \approx -1.1$ and $\xi_y \approx -0.3$. (d) $Q_x = 6.21$, $Q_y = 6.16$, $\xi_x \approx -1.2$ and $\xi_y = 0.1$. (e) Ultimate single-bunch beam with $Q_x = 6.20$, $Q_y = 6.16$, $\xi_x \approx -1.3$ and $\xi_y = 0.1$.

R. Cappi, R. Garoby, E. Métral, AIP proc. 496 (1999)

■ Barrier Bucket / one turn constant wake

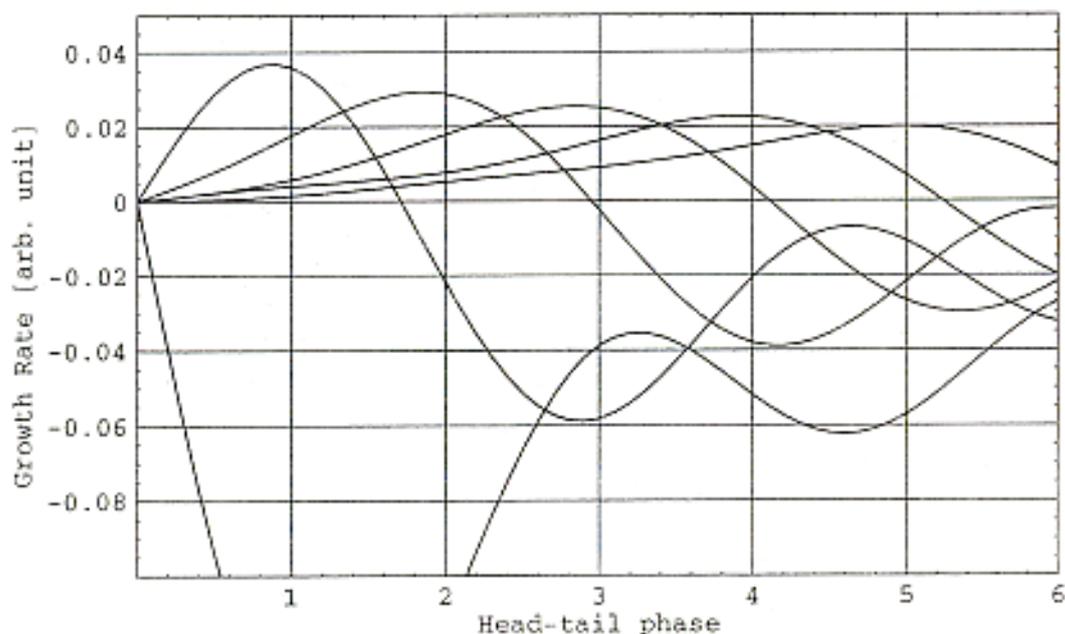
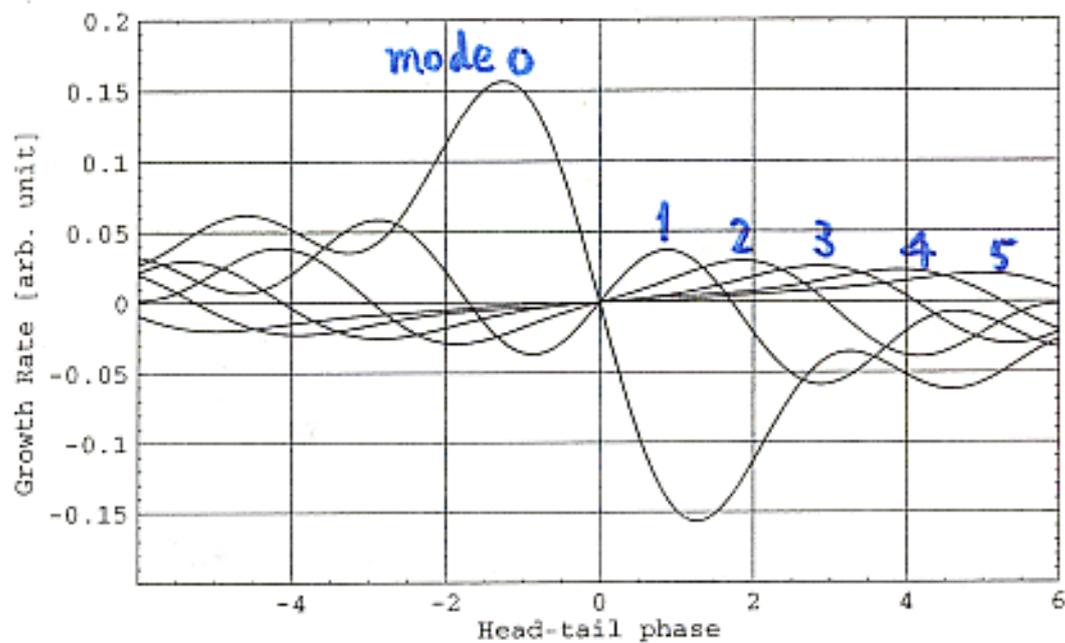


Stable →
← unstable



$$\frac{\xi \omega \beta \hat{z}}{c \eta}$$

■ Harmonic RF / one-turn constant wake



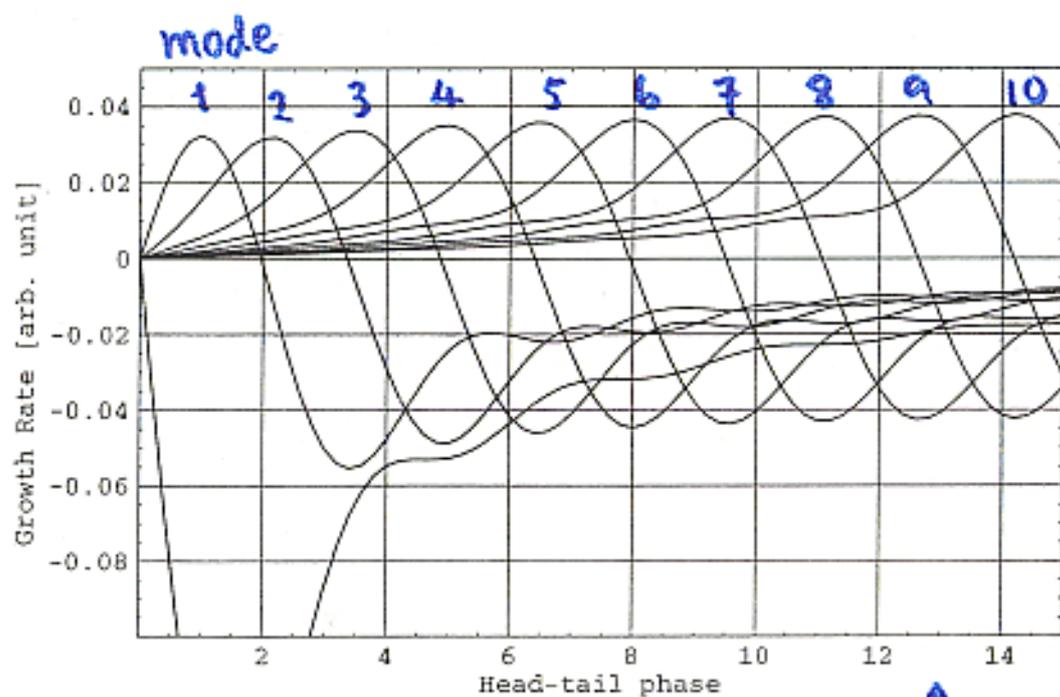
$$\text{Head-tail phase} = \frac{\xi \omega_{\beta} \hat{z}}{c \eta}$$

ξ : chromaticity,

ω_{β} : betatron freq.

\hat{z} : bunch half length,

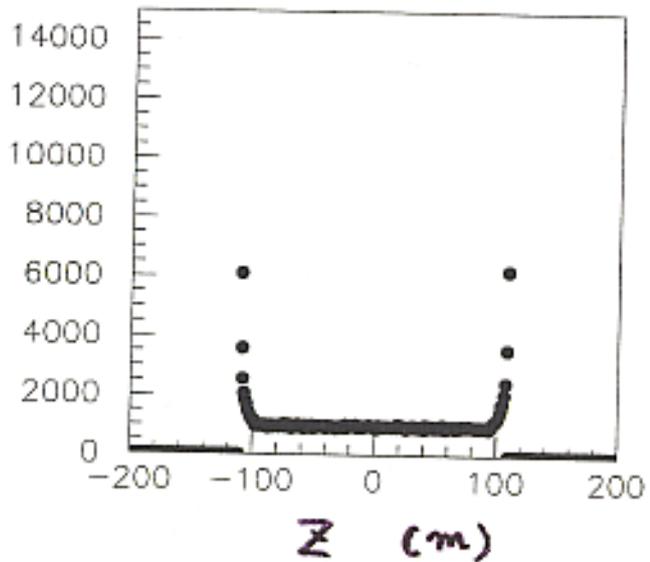
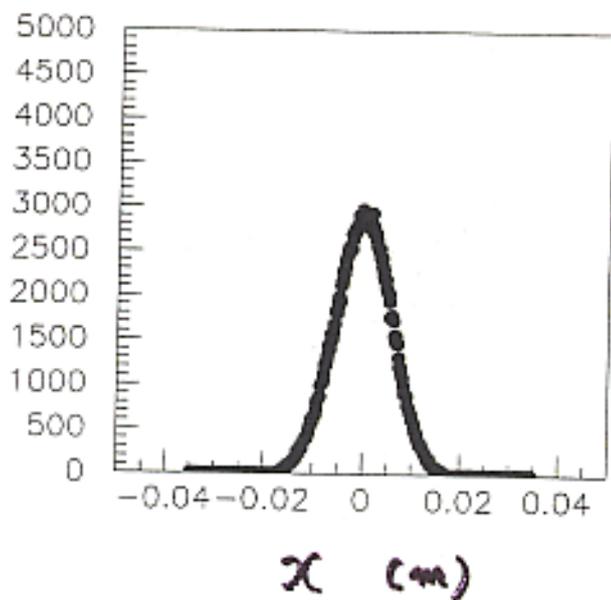
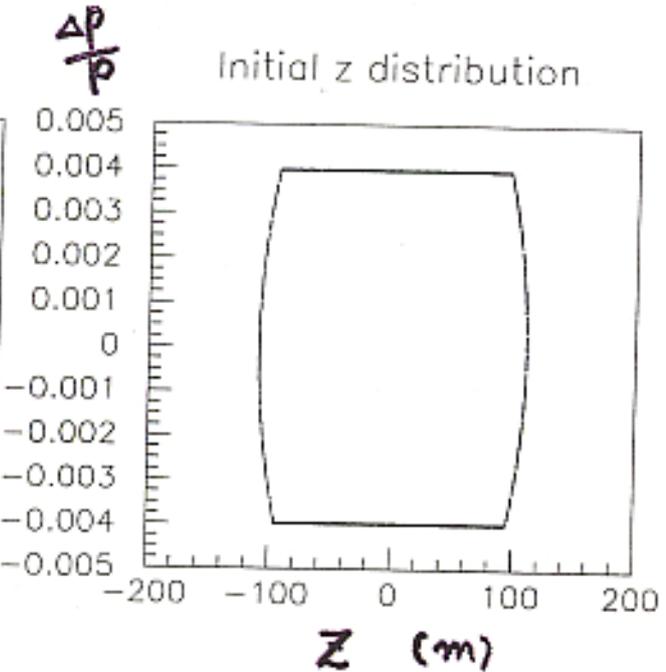
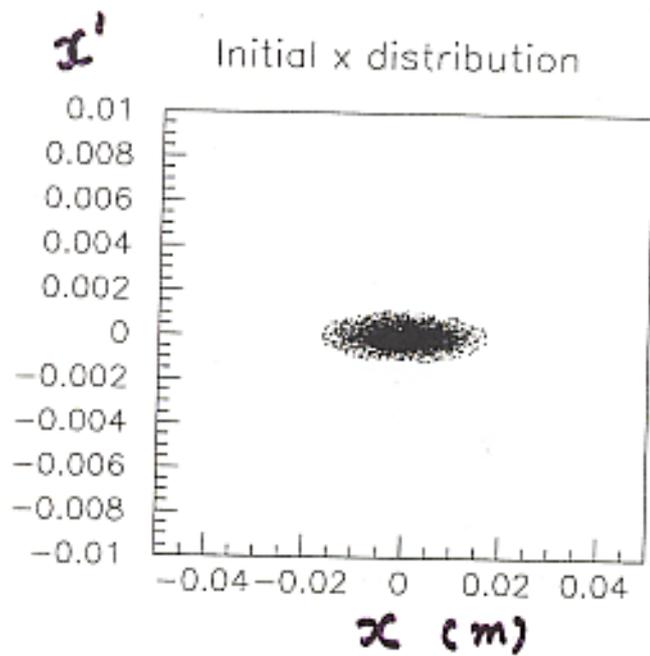
η : slippage factor



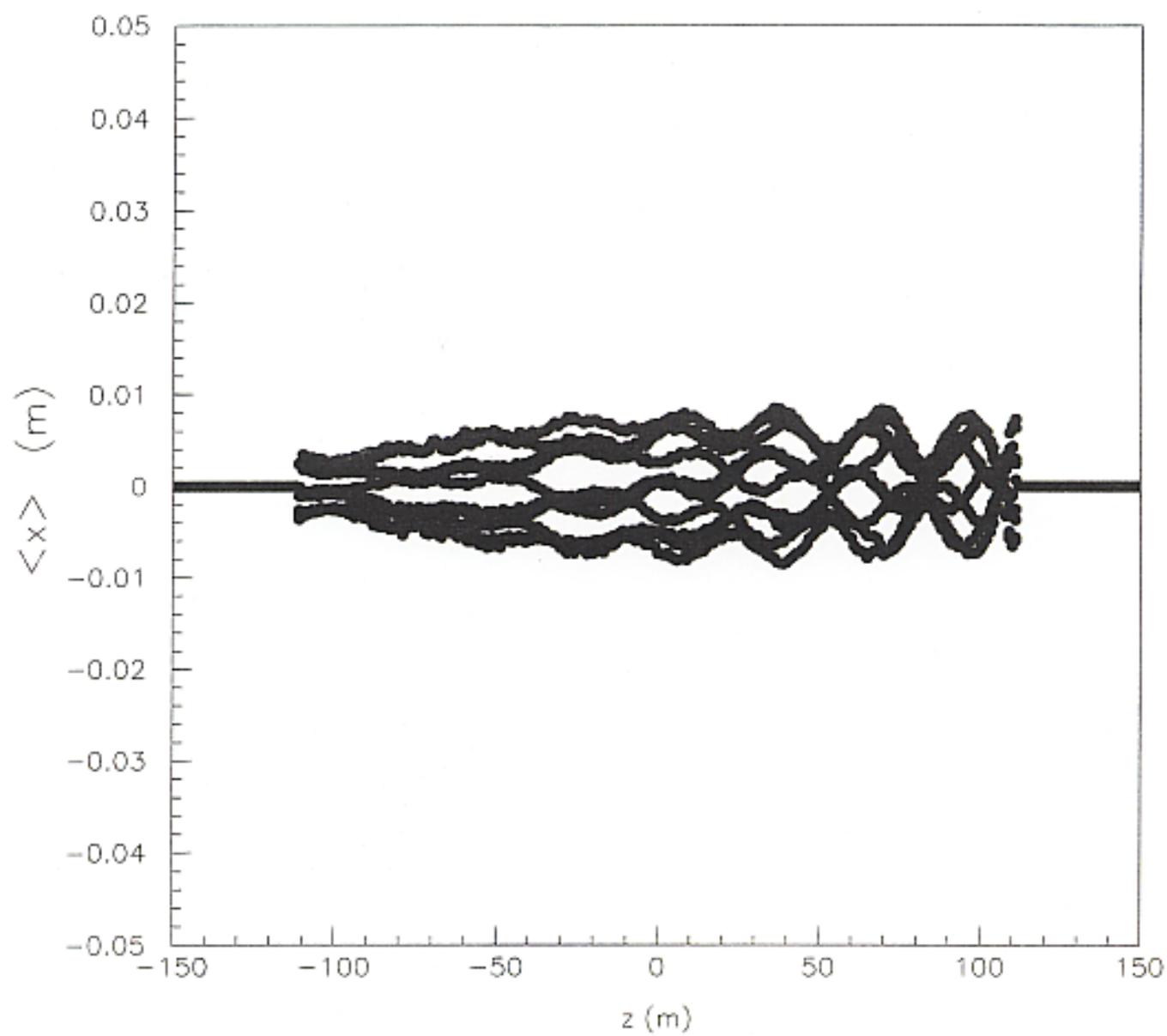
↑
simulation

$$\xi = -2$$
$$2\hat{z} \approx 200 \text{ m}$$

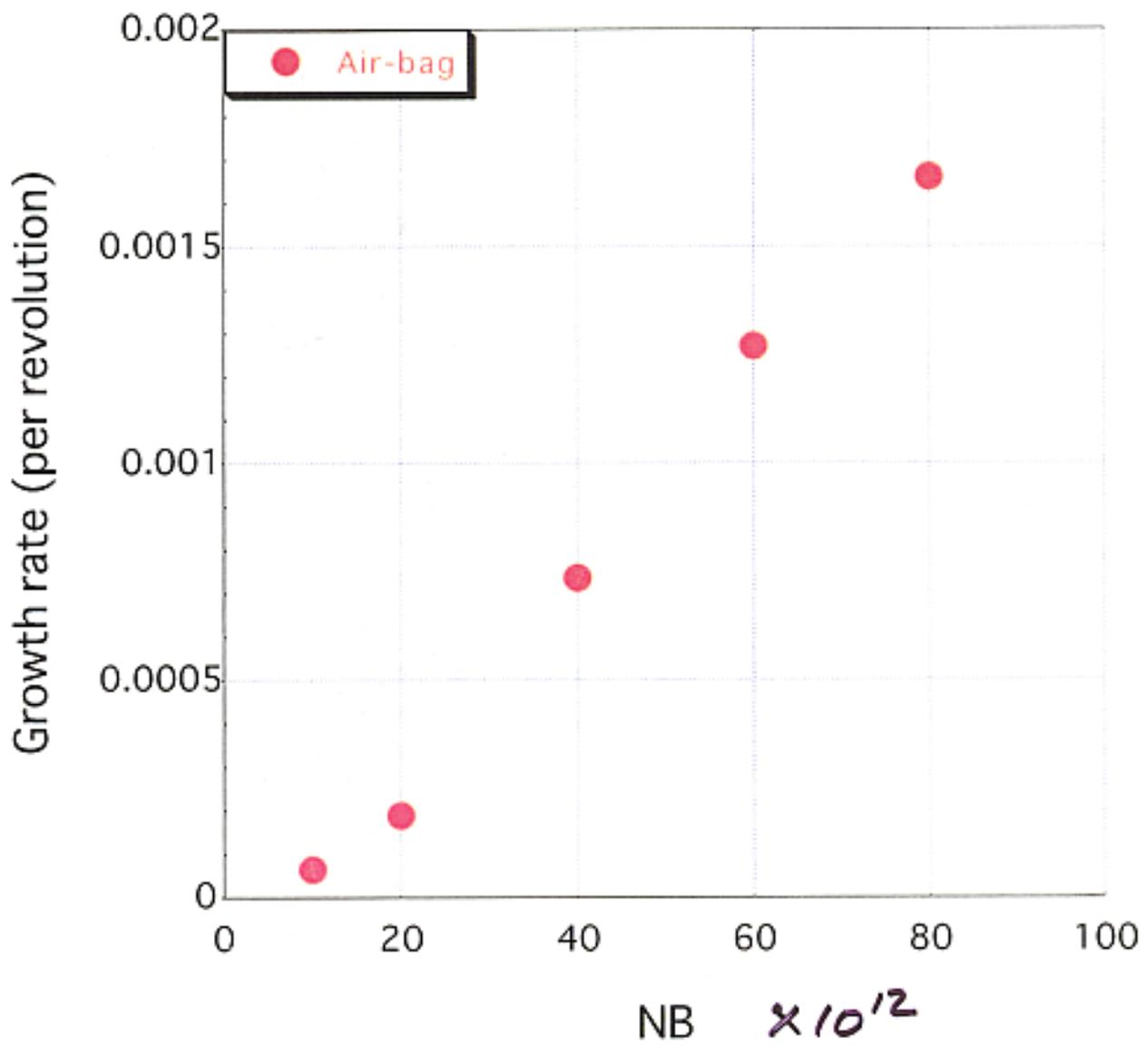
Air-bag model



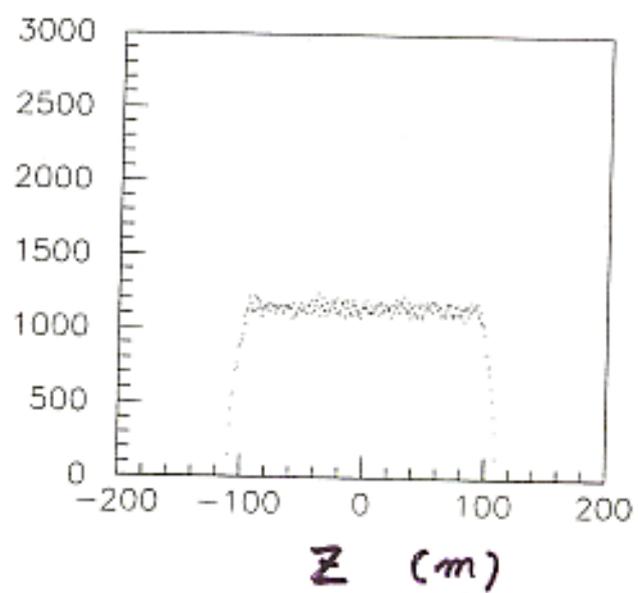
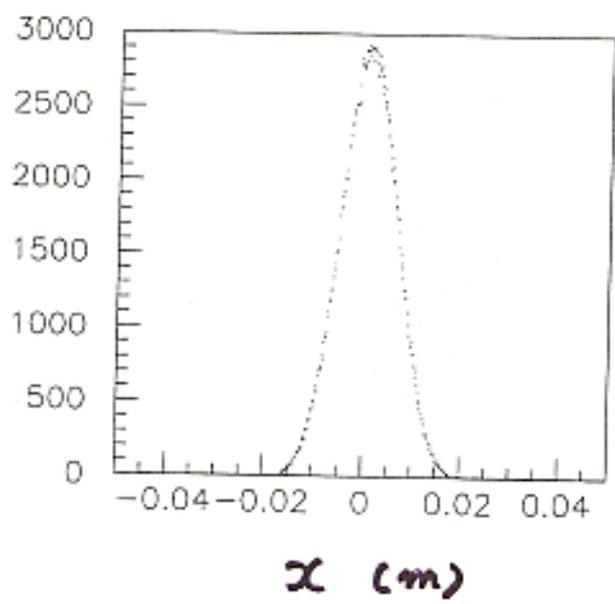
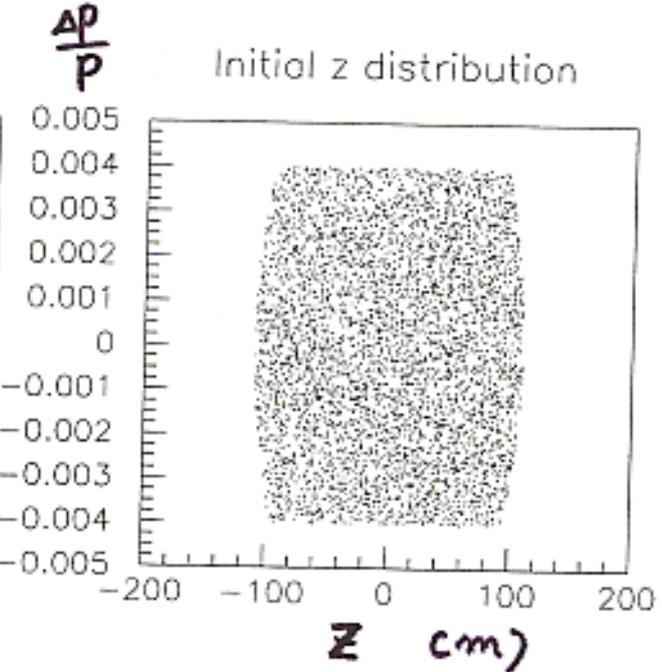
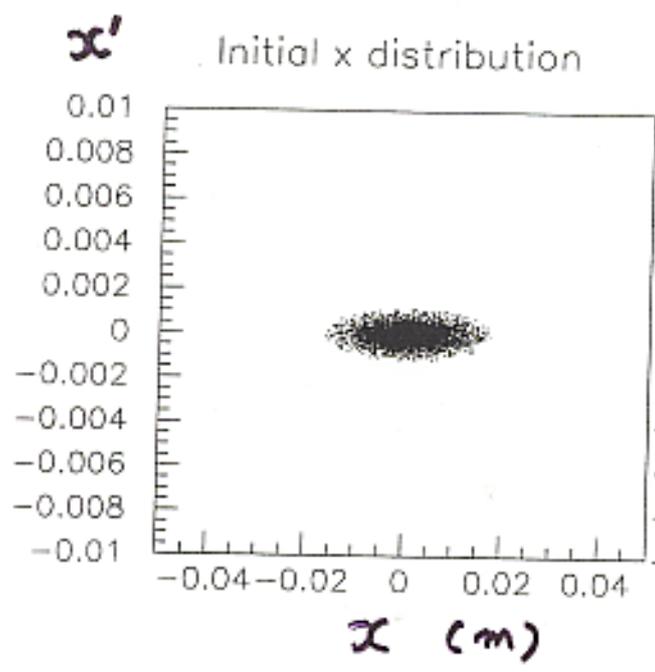
Dipole oscillation $\langle x \rangle$ vs z



Superbunch / one-turn constant wake $W_0 = 10^{12} \frac{V}{C}$

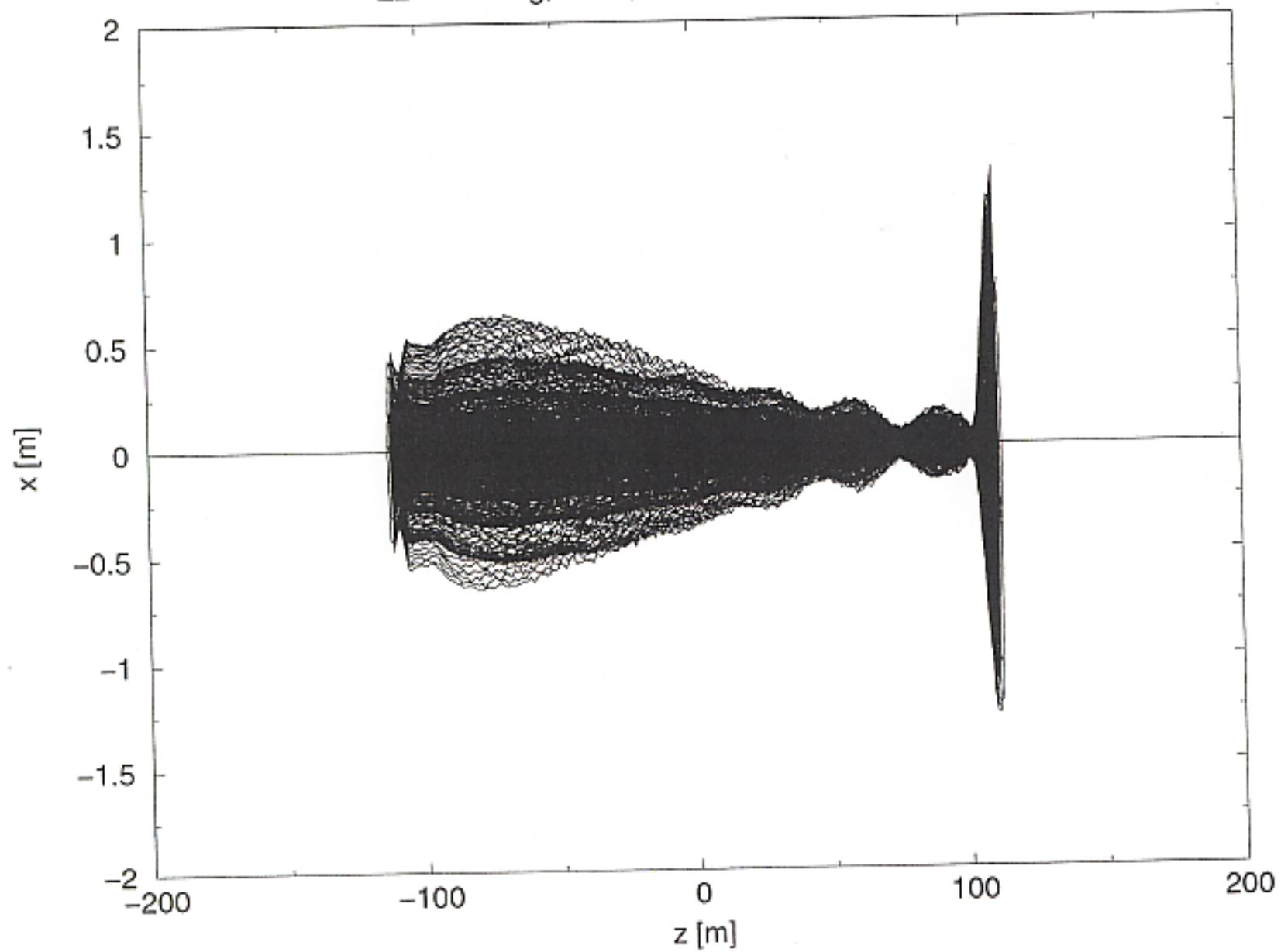


Water-bag model

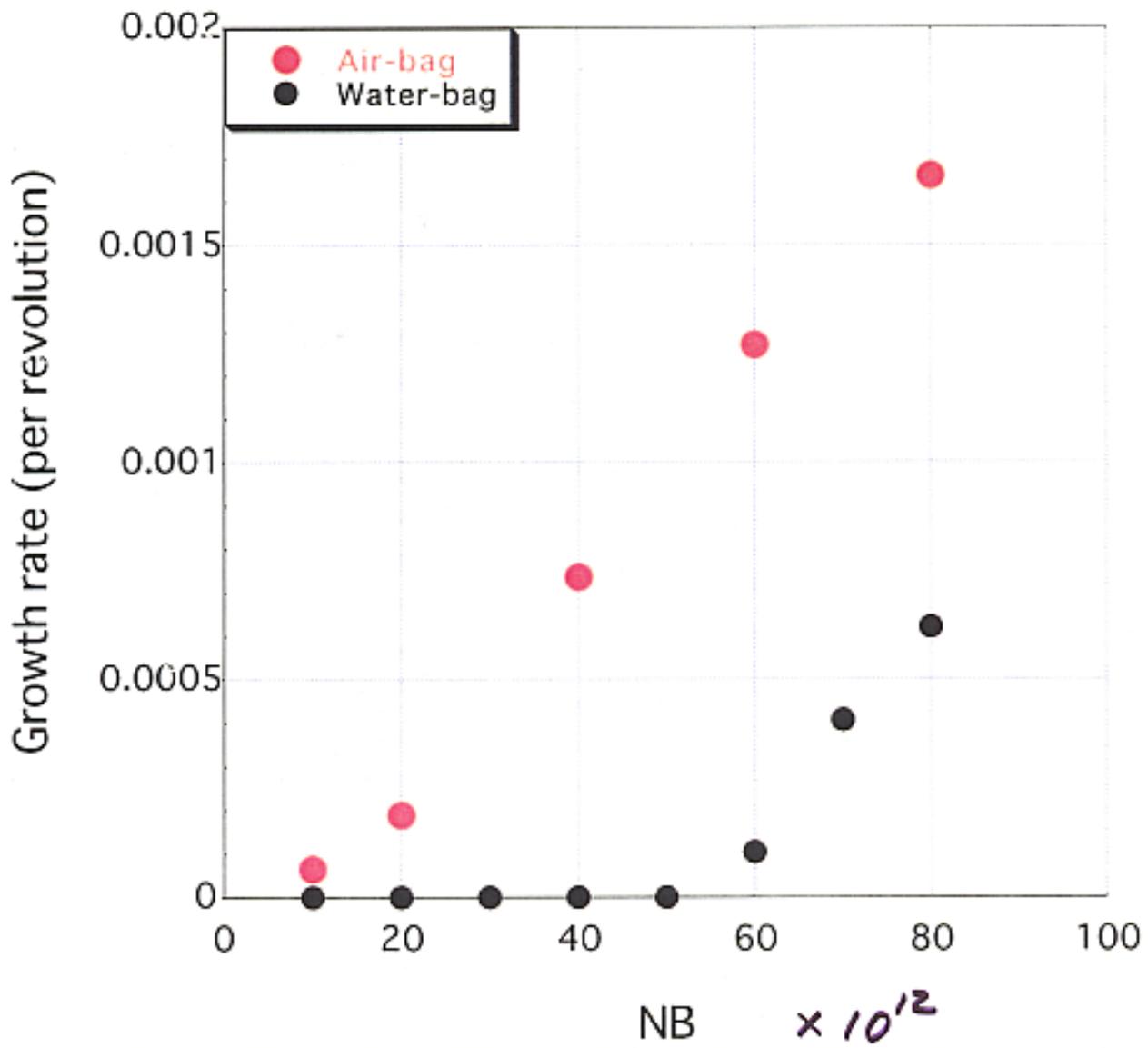


Superbunch

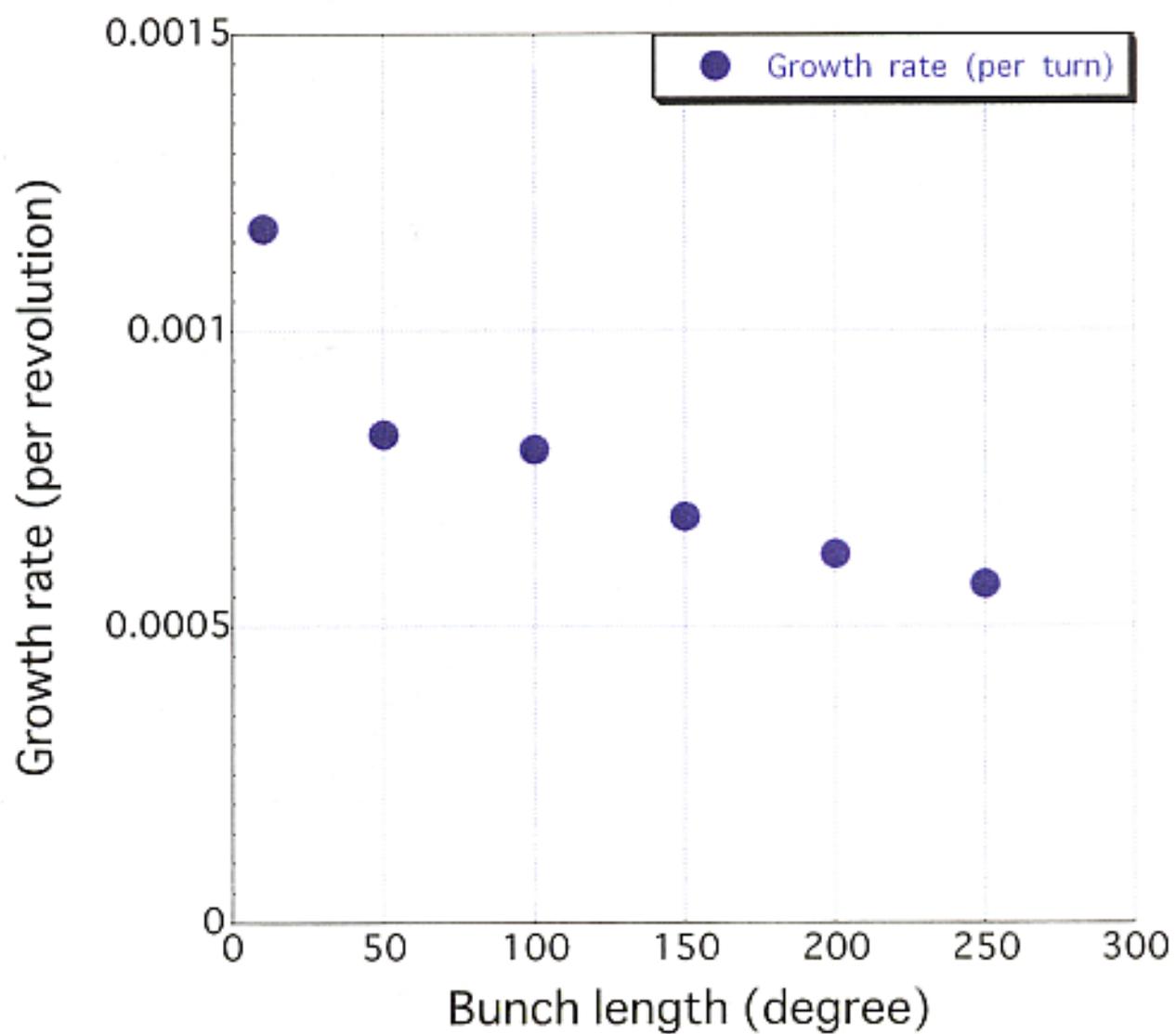
Z2->200deg, 60kV, NB=80, W0=10**12, xi=-2



Superbunch / one-turn constant wake $W_0 = 10^{12} \frac{V}{C.m}$



Superbunch / one-turn constant wake



Summary

- ◆ Transverse head-tail instability with superbunch
 - ◆ **Air-bag model**
 - ◆ with $\xi < 0$, higher order modes are unstable.
 - ◆ with $\xi > 0$, 0-th order modes are unstable.
 - ◆ **Water-bag model**
 - ◆ Unstable higher order modes in the air-bag model are suppressed by synchrotron tune spread vs. vs ~ 0.0011 in this case
Growth rate of ~ 0.001 may be suppressed by ns
 - ◆ Longer bunch tends to be more stable, if total NB is fixed.

Superbunch seems to add no difficulty as far as transverse instability.

Experiment of Head-tail Instability using an RF barrier-bucket is expected soon. It may be possible by controlling chromaticity.