

Beam-Beam Effects and Longitudinal Dynamics etc. for LHC Upgrade based on Superbunches

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- beam-beam & luminosity:
 - long range for bunches & superbunches, x - y & $45^\circ/135^\circ$
 - tune shift, tune spread, tune footprints, tune diffusion, amplitude diffusion, coupling
- electron cloud
 - build up & instability
- intrabeam scattering
- barrier bucket dynamics with synchrotron radiation

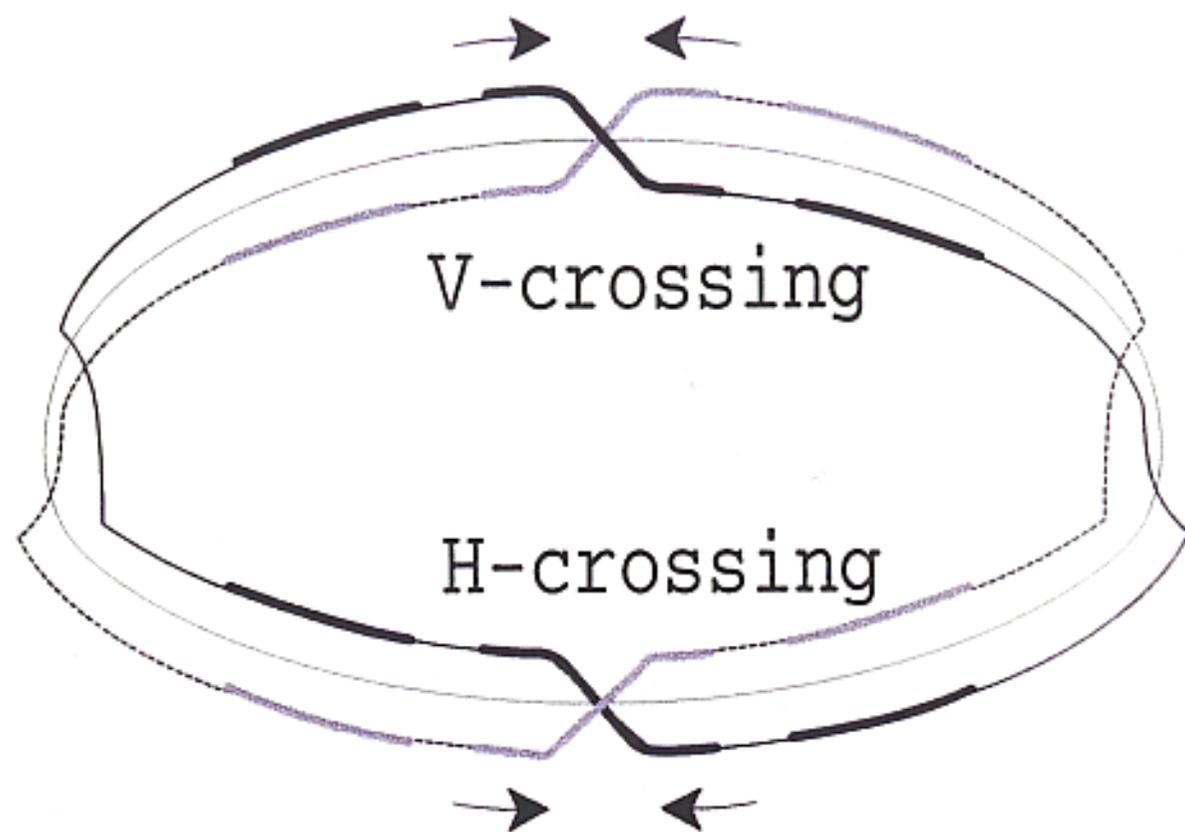
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Superbunches - An Alternative Approach for Proton Colliders:

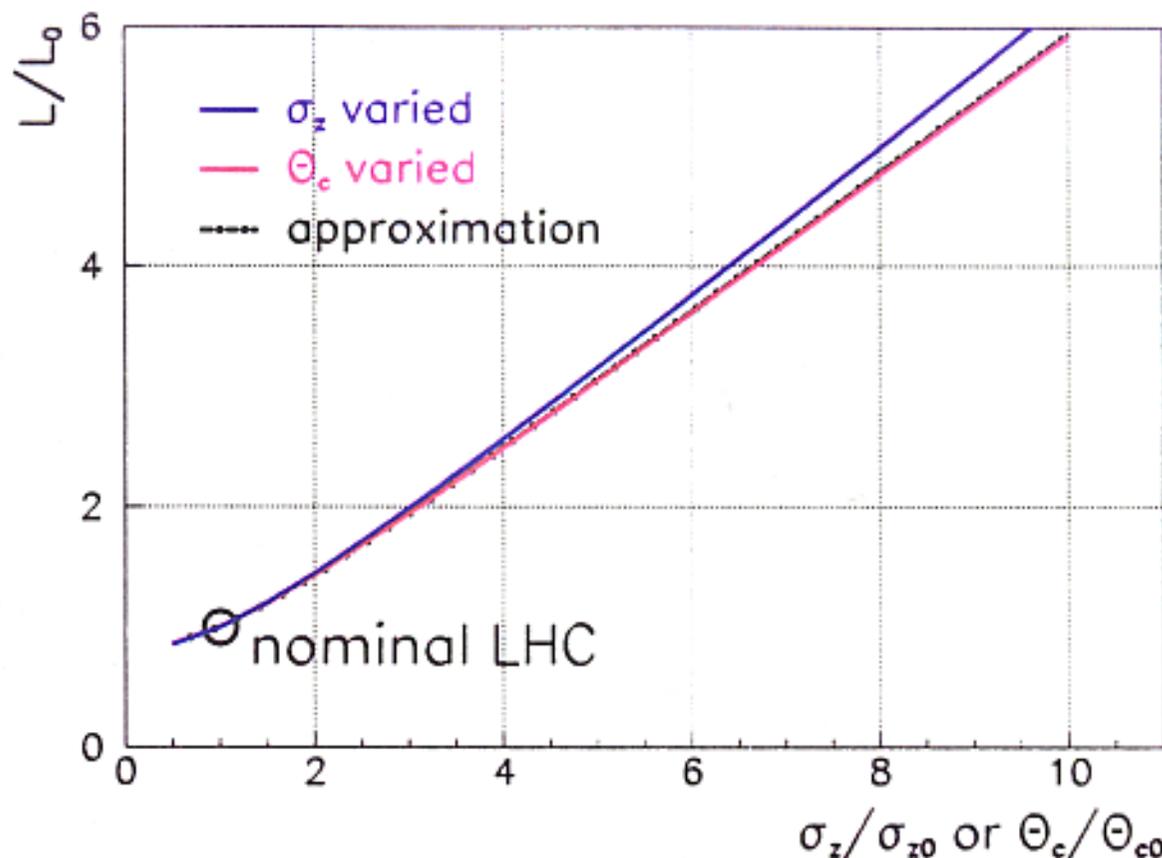
(K. Takayama et al.)

LHC Upgrade using 'Continuous Beams' or Super Bunches?

- ISR experience
- higher luminosity!
- negligible heat load from e^- cloud!
- no PACMAN bunches



Schematic of **Super Bunches** in a High-Luminosity Collider (K. Takayama et al., Snowmass 2001) Alternating crossing in ' x/y ' or $45^\circ/135^\circ$ ('**hybrid**') planes



Relative increase in LHC luminosity vs. bunch length (or crossing angle), maintaining a constant beam-beam tune shift with alternating crossing; for $\sigma_{z0} = 7.8$ cm, $\theta_{c0} = 300 \mu\text{rad}$, $\sigma^* = 16 \mu\text{m}$, $\beta^* = 0.5$ m, $L_0 = 10^{34} \text{ cm}^{-2}\text{s}^{-1}$. (F.Z. & F.R., PRSTAB 5, 061001, 2002)

Nominal and ultimate LHC parameters
 compared with LHC superbunch upgrade.

no. of bunches	2808	2808	2808	1
bunch pop. [10^{11}]	1.1	1.67	2.6	5600
rms bunch length (σ_z) [cm]	7.7	7.7	7.7	7500
energy spread (σ_δ) [10^{-4}]	1.1	1.1	1.1	5.8
beta-star [m]	0.5	0.5	0.25	0.25
crossing angle [μ rad]	300	300	466	1000
beam current [A]	0.56	0.86	1.32	1.0
luminosity [10^{34} cm $^{-2}$ s $^{-1}$]	1	2.3	7.3	9.0
IBS growth time $\tau_{z,IBS}$ [h]	67	43	28	856

the total tune shift for two IPs is the same for conventional alternating crossing (x and y) and for the hybrid scheme, namely

$$\Delta Q_{\text{tot}} = \Delta Q_x + \Delta Q_y = + \frac{\lambda r_p \beta^*}{\pi \gamma} \left(\frac{1 + \cos \theta}{2} \right) \int_{-l/2}^{l/2} \left(1 + \frac{s^2}{\beta^{*2}} \right) \left[\frac{\cos \theta - 1}{s^2 \sin^2 \theta} \left(1 - \exp \left(-\frac{s^2 \sin^2 \theta}{2\sigma^2} \right) \right) - \exp \left(-\frac{s^2 \sin^2 \theta}{2\sigma^2} \right) \frac{\cos \theta}{\sigma^2} \right] ds ,$$

(superbunches)

(F.Z. & F.R., PRSTAB 5, 061001 (2002))

analogous expression for Gaussian bunches applies to both $x - y$ and hybrid crossings:

$$\Delta Q_{\text{tot}} = -\frac{\lambda r_p \beta^*}{\pi \gamma} \left(\frac{1 + \cos \theta}{2} \right) \int_{-l/2}^{l/2} (1 - \cos \theta) \left(1 + \frac{s^2}{\beta^{*2}} \right) \left[\frac{1}{s^2 \sin^2 \theta} \left(1 - \exp \left(-\frac{s^2 \sin^2 \theta}{2\sigma^2} \right) \right) + \cos \theta \exp \left(-\frac{s^2 \sin^2 \theta}{2\sigma^2} \right) \frac{1}{\sigma^2} + \frac{1 + \cos \theta}{\sigma_z^2} \left(1 - \exp \left(-\frac{s^2 \sin^2 \theta}{2\sigma^2} \right) \right) \right] g(s) ds \quad (\text{Gaussian bunch}) . \quad (1)$$

where $g(s)$ is the Gaussian form factor

$$g(s) = \exp \left(-\frac{s^2 (1 + \cos \theta)^2}{2\sigma_z^2} \right)$$

(F.Z. & F.R., PRSTAB 5, 061001 (2002))

Simplified Expressions: Luminosity and Beam-Beam Tune Shift for Gaussian Bunches (2 IPs with alt. Xing)

assume $\theta \ll 1$, $\sigma^* \ll \sigma_z \ll \beta^*$ and $\theta\sigma_z/(2\sigma^*) \gg 1$

$$\Delta Q_{\text{tot}} \approx -\frac{N_b r_p \beta^*}{2\pi\gamma\sigma^* \sqrt{\sigma^{*2} + \theta^2\sigma_z^2/4}} \approx \sqrt{\frac{2}{\pi}} \frac{r_p \beta^*}{\gamma\sigma^* \theta} \hat{\lambda}$$

$$L \approx \frac{f_{\text{coll}} N_b^2}{4\pi\sigma^* \sqrt{\sigma^{*2} + \sigma_z^2\theta^2/4}} \approx \frac{f_{\text{coll}} N_b^2}{2\pi\sigma^* \sigma_z \theta}$$

combine these to

$$\begin{aligned} L &\approx \frac{f_{\text{coll}} \gamma \epsilon_n}{r_p^2 \beta^*} \Delta Q_{\text{tot}}^2 \pi \sqrt{1 + \frac{\theta^2 \sigma_z^2}{4\sigma^{*2}}} \\ &\approx \frac{f_{\text{coll}} \gamma \epsilon_n}{r_p^2 \beta^*} \Delta Q_{\text{tot}}^2 \frac{\pi \theta \sigma_z}{2\sigma^*} \end{aligned}$$

Simplified Expressions: Luminosity and 2-IP Beam-Beam Tune Shift for Superbunches

assume $\sqrt{\epsilon_n/(\gamma\beta^*)} \ll \theta \ll 1$ and $l_{\text{bunch}} \gg 10\sigma^*/\theta$

then, from Eq. (18) in our PRST-AB paper

$$\Delta Q_{\text{tot}} \approx -\frac{\lambda r_p \sqrt{2}}{\sqrt{\pi} \gamma} \sqrt{\frac{\beta^* \gamma}{\epsilon_n}} \frac{1}{\theta} \text{Erf} \left(\frac{l\theta}{2\sqrt{2}\sigma^*} \right) \approx -\frac{\lambda r_p \sqrt{2}}{\sqrt{\pi} \gamma} \sqrt{\frac{\beta^* \gamma}{\epsilon_n}} \frac{1}{\theta},$$

and Eq. (32) simplifies to

$$L = \frac{f_{\text{coll}} l_{\text{bunch}} \lambda \gamma}{2\pi} \left(\frac{\lambda}{\epsilon_n} \right) \int_{-l_{\text{det}}/(2\beta^*)}^{l_{\text{det}}/(2\beta^*)} \frac{1}{1+u^2} \exp \left[-\frac{\beta^{*2} \theta^2}{4\sigma^{*2}} \frac{u^2}{1+u^2} \right] du$$

$$\approx \frac{f_{\text{coll}} l_{\text{bunch}} \lambda^2}{\sqrt{\pi}} \frac{1}{\theta} \frac{1}{\sqrt{\beta^* \epsilon_n / \gamma}} \quad \text{combinetheseto}$$

$$L \approx \frac{\gamma \epsilon_n}{r_p^2 \beta^*} f_{\text{coll}} \Delta Q_{\text{tot}}^2 \left(\frac{\sqrt{\pi} \theta l_{\text{bunch}}}{2 \sigma^*} \right)$$

we find the correspondence $l_{\text{bunch}} \simeq \sqrt{\pi} \sigma_z!$

Why is the luminosity higher for superbunches?

rewrite tune shifts as

$$\Delta Q_{\text{tot}} \approx \sqrt{\frac{2}{\pi}} \frac{r_p \beta^*}{\gamma \sigma^* \theta} \hat{\lambda} \quad (\text{Gaussian})$$

$$\Delta Q_{\text{tot}} \approx \sqrt{\frac{2}{\pi}} \frac{r_p \beta^*}{\gamma \sigma^* \theta} \lambda \quad (\text{superbunch})$$

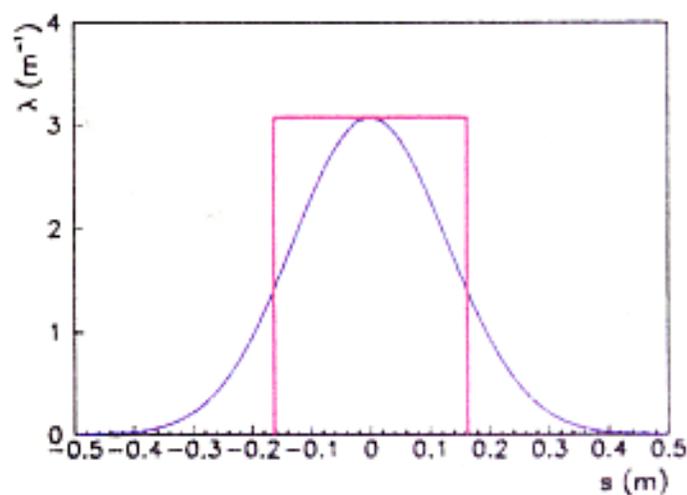
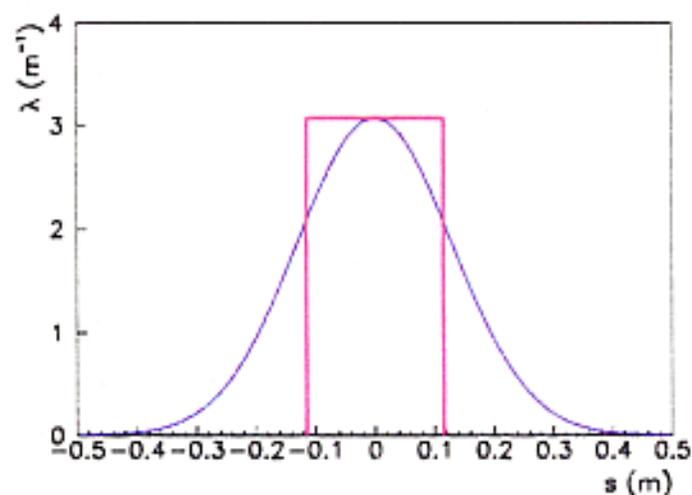
then re-express luminosity as

$$L \approx \frac{1}{2} \frac{f_{\text{coll}}}{r_p} \frac{\Delta Q_{\text{tot}} N_b \gamma}{\beta^*} \quad (\text{Gaussian})$$

$$L \approx \frac{1}{\sqrt{2}} \frac{f_{\text{coll}}}{r_p} \frac{\Delta Q_{\text{tot}} N_b \gamma}{\beta^*} \quad (\text{superbunch})$$

for the same total current and total tune shift, luminosity of superbunches is $\sqrt{2}$ times higher!

to be precise, the luminosity is simply higher for flat bunches compared with Gaussians!!

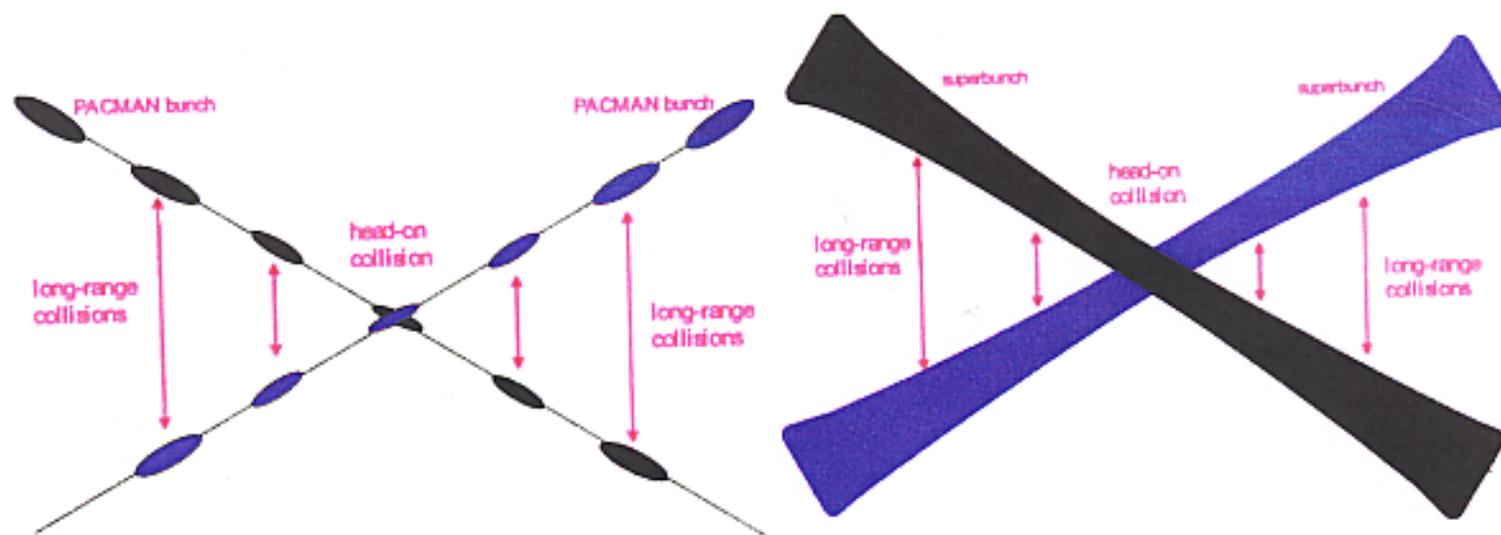


profiles that give equal tune shifts and either equal luminosity (left) or the same total current and $\sqrt{2}$ higher luminosity (right)

there are several ways to generate a flat beam

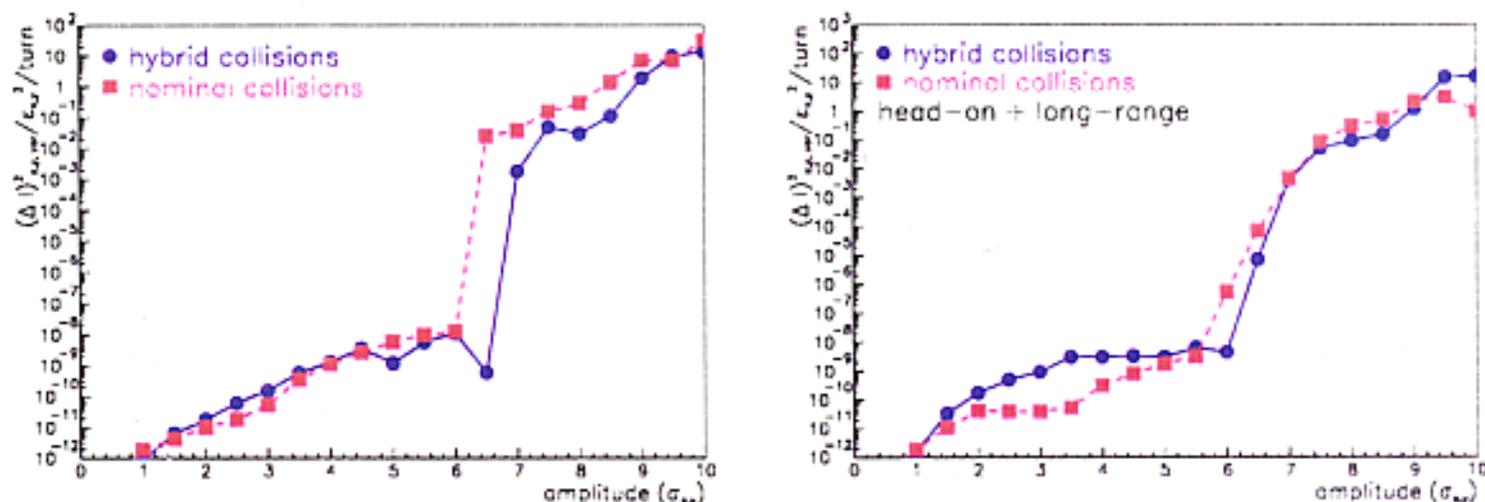
- **superbunch in barrier bucket** (K. Takayama, et al., KEK PS, FNAL?)
- introduce **empty phase space in the centre of coasting beam** before bunching (J.P. Delahaye, et al., 1980; A. Blas et al., 2000; CERN PS Booster)
- **recombination with an empty bucket** (C. Carli, M. Chanel; CERN PS Booster)
- **redistribution of surfaces in phase space** (C. Carli, M. Chanel; CERN PS Booster)

Long-Range & Superbunch Collisions



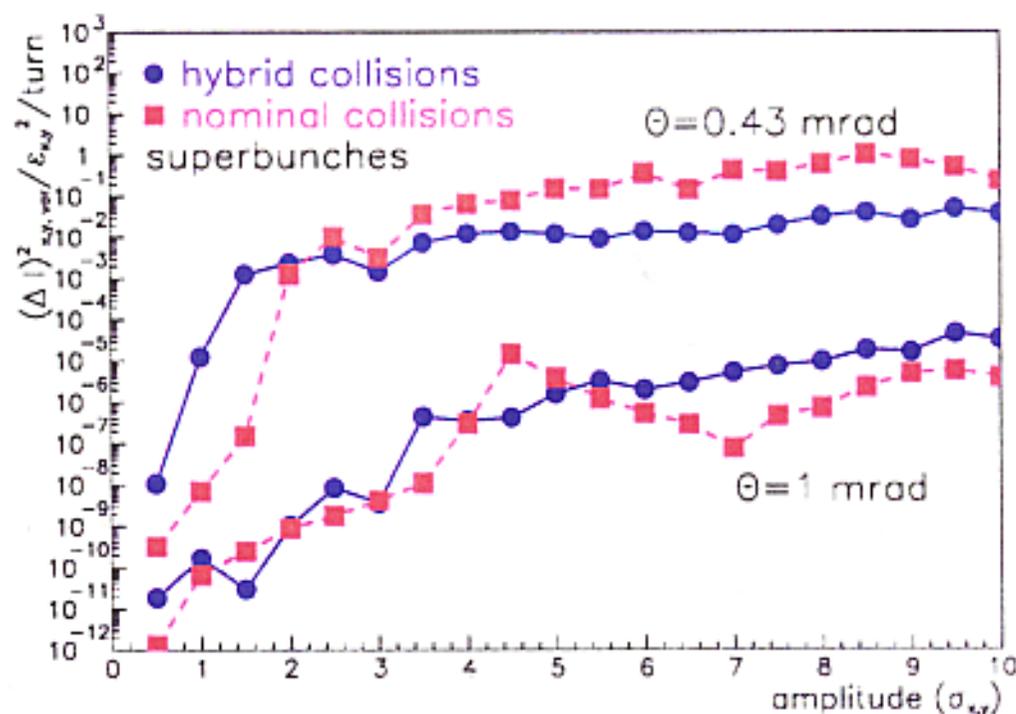
Left: Gaussian bunches (LHC); right: superbunches (LHC-II?)

Amplitude Diffusion Rate for Gaussian Bunches



Simulated diffusion rate vs. start amplitude with $x - y$ alternating crossing and with hybrid collisions; $\beta^* = 0.5$ m, $\theta_{x,y}^* = 31.7 \mu\text{rad}$, $\theta_c = 300 \mu\text{rad}$; nominal LHC. left: long-range collisions only; right: long-range & head-on collisions (Y.P. & F.Z., PRST-AB 2 104001, 1999)

Amplitude Diffusion Rate for Superbunches



Simulated diffusion rate vs. start amplitude with $x - y$ alternating crossing & hybrid LHC-II superbunch collisions, $\beta_{x,y}^* = 0.25 \text{ m}$, $\theta_c = 1 \text{ mrad}$ & $\theta_c = 426 \mu\text{rad}$, $\theta_{x,y}^* = 44.8 \mu\text{rad}$, $\lambda = 8.8 \times 10^{11} \text{ m}^{-1}$, $l = 2 \text{ m}$, 100 steps/collision.

Tune spread can also be computed, in 1st order perturbation theory, starting from beam-beam potential in action-angle variables

$$U(J_x, J_y, \phi_x, \phi_y, s_{\max}, \theta) = \frac{(1 + \cos \theta) r_p \lambda}{\pi \gamma} \int_{-l/2}^{s_{\max}} ds \int_{\infty}^{R(s)} \frac{dw}{w} \left(1 - \exp\left(-\frac{w^2}{2}\right) \right)$$

with

$$R(s) \equiv \left(\frac{(x \cos \theta - s \sin \theta)^2 + y^2}{\epsilon \beta} \right)^{1/2}$$

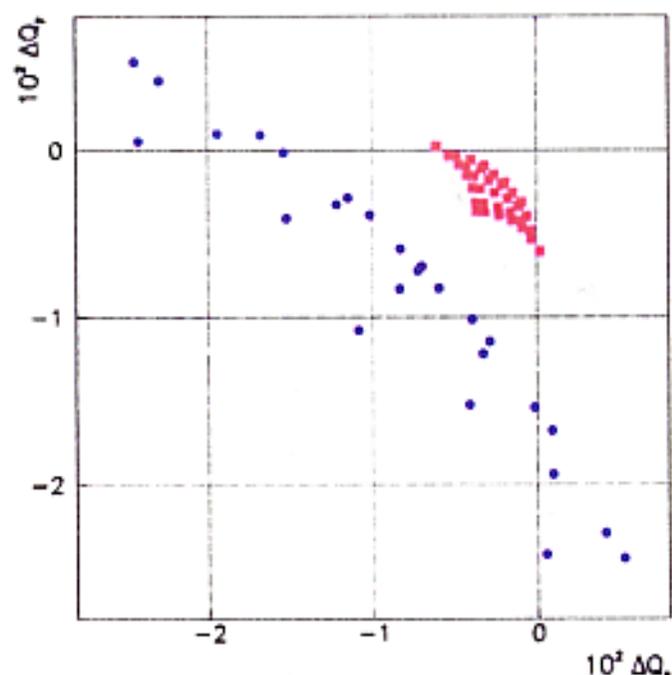
$$x = \sqrt{2J_x \beta_x} \cos \phi_x$$

$$y = \sqrt{2J_y \beta_y} \cos \phi_y$$

and evaluates

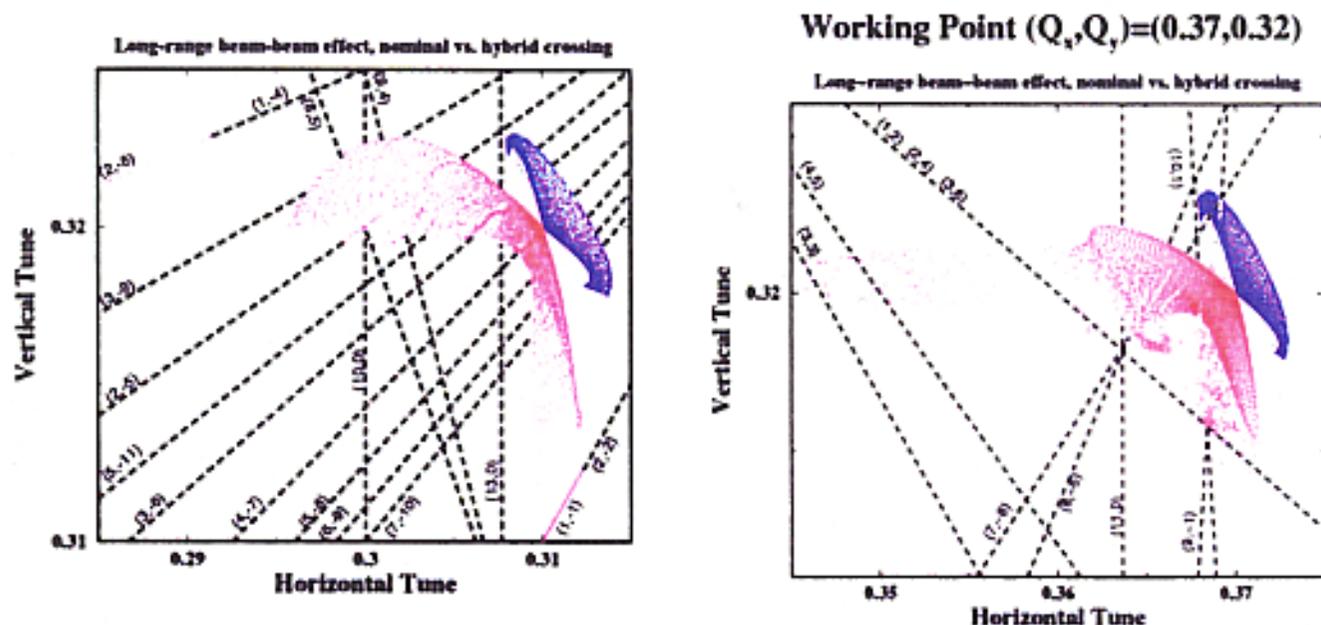
$$\Delta Q_{x,y} = \left\langle \frac{\partial U}{\partial J_{x,y}} \right\rangle_{\phi_x, \phi_y} \quad (2)$$

Typical tune footprints obtained by solving Eq. (2) numerically



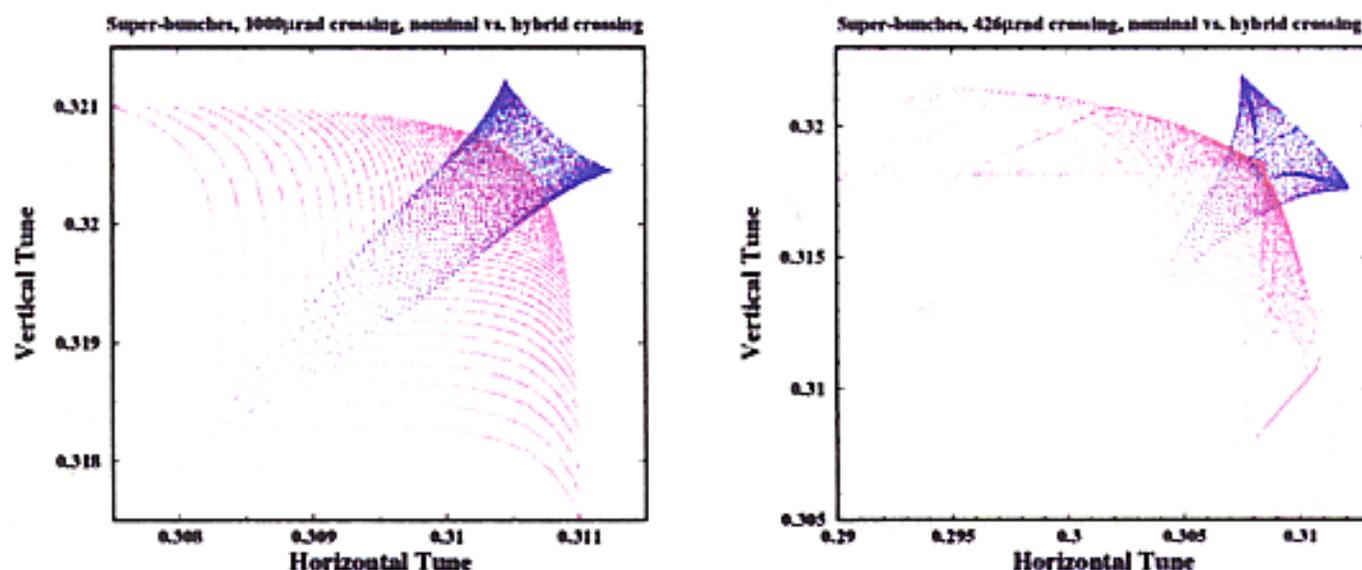
Tune footprints for two different crossing angles: $\theta_c = 400 \mu\text{rad}$ (blue circles) and $\theta_c = 1 \text{ mrad}$ (red squares), and betatron amplitudes extending from 0 to 6σ . Other parameters: $\lambda = 8.8 \times 10^{11} \text{ m}^{-1}$, $\beta_{x,y}^* = 0.25 \text{ m}$, $l = 40 \text{ m}$. (F.Z. & F.R., PRSTAB 5, 061001, 2002)

Simulated Tune Footprints (Gaussian Bunches)



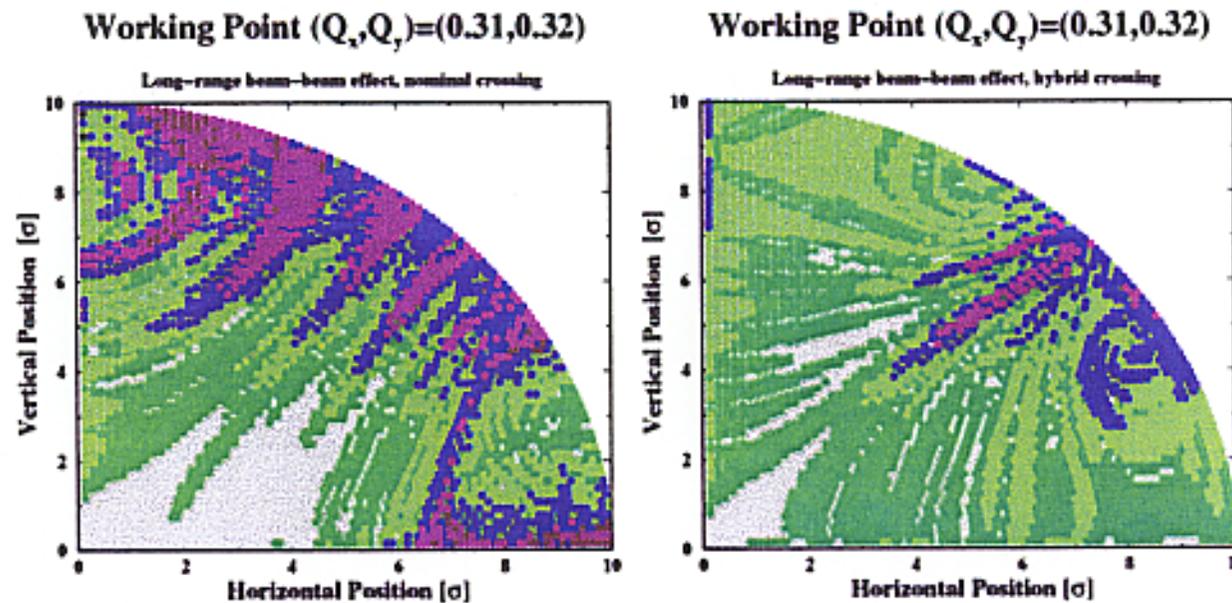
Tune footprints from a frequency-map analysis a la Laskar for alternating $x - y$ and hybrid crossings; long-range collisions only; left: $Q_x = 0.31$, $Q_y = 0.32$ (nominal); right: $Q_x = 0.37$, $Q_y = 0.32$ (Y. Papaphilippou)

Tune Footprints (Superbunches)



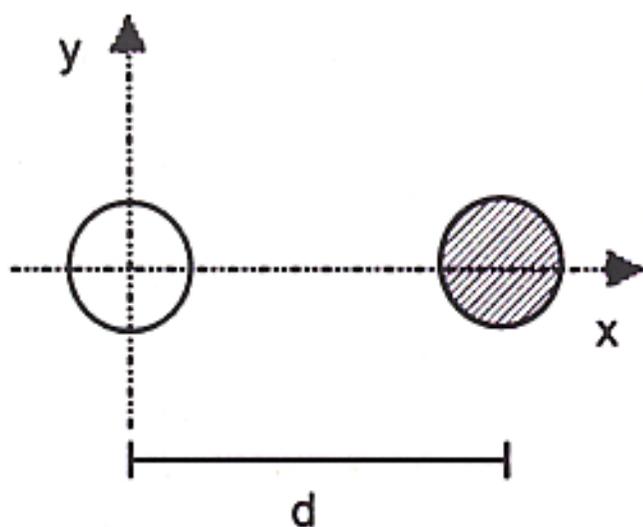
Tune footprints from a frequency-map analysis a la Laskar for $x - y$ or hybrid crossing of superbunches, and $\lambda = 8.8 \times 10^{11} \text{ m}^{-1}$. left: $\theta_c = 1.0 \text{ mrad}$; right: $\theta_c = 0.426 \text{ mrad}$ (Y. Papaphilippou)

Tune Diffusion (Gaussian Bunches)



Tune diffusion maps for alternating $x - y$ (left) and hybrid (right) long-range crossings at two IPs and the base tunes $Q_x = 0.31$, $Q_y = 0.32$. color indicates tune difference between turns 0...1000 and 1000...2000, on a logarithmic scale, from less than 10^{-7} (gray) to greater than 10^{-2} (black). (Y. Papaphilippou)

A Simpler Look at Tune Shift for $x - y$ Crossing



horizontal crossing \rightarrow
 (~~de~~)focusing quadrupole of
 strength

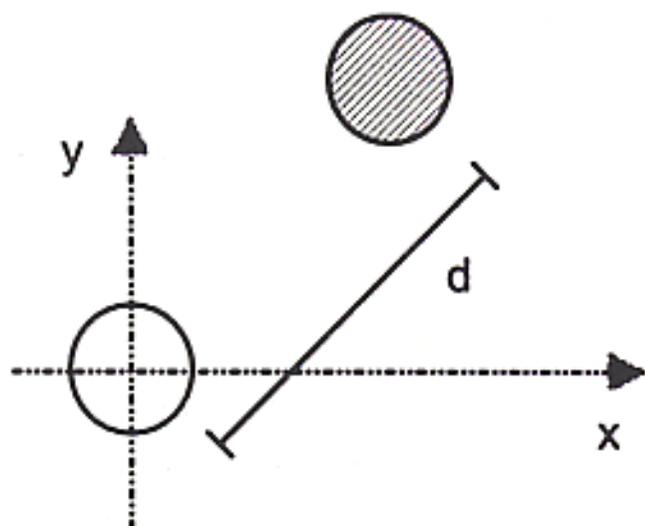
$$K = \frac{2N_b r_p}{\gamma^2 d^2}$$

$$\Delta x' = \frac{2N_b r_p (x+d)}{\gamma((d+x)^2 + y^2)} \equiv C \frac{(-x)}{(d+x)^2 + y^2}$$

$$\Delta y' = \frac{2N_b r_p y}{\gamma((d+x)^2 + y^2)} \equiv C \frac{y}{(d+x)^2 + y^2}$$

$$\left. \frac{\partial \Delta x'}{\partial x} \right|_{x=y=0} = -C \frac{1}{d^2}, \quad \left. \frac{\partial \Delta y'}{\partial y} \right|_{x=y=0} = C \frac{1}{d^2}$$

Betatron Coupling from Hybrid Crossing



45 or 135 degree crossing \rightarrow skew quadrupole of strength

$$K_s = \pm \frac{2N_b r_p}{\gamma^2 d^2}$$

$$\frac{\partial \Delta x'}{\partial y} = -C \frac{1}{d^2}, \quad \frac{\partial \Delta y'}{\partial x} = -C \frac{1}{d^2} \quad (\text{for } 45^\circ)$$

$$\frac{\partial \Delta x'}{\partial y} = C \frac{1}{d^2}, \quad \frac{\partial \Delta y'}{\partial x} = C \frac{1}{d^2} \quad (\text{for } 135^\circ)$$

Closest Tune Approach (Coupling Strength)

$$|\kappa_-| = \left| \frac{1}{2\pi} \oint ds K_s \sqrt{\beta_x \beta_y} e^{i(\phi_x - \phi_y - (Q_x - Q_y - q) \frac{2\pi s}{L})} \right|$$

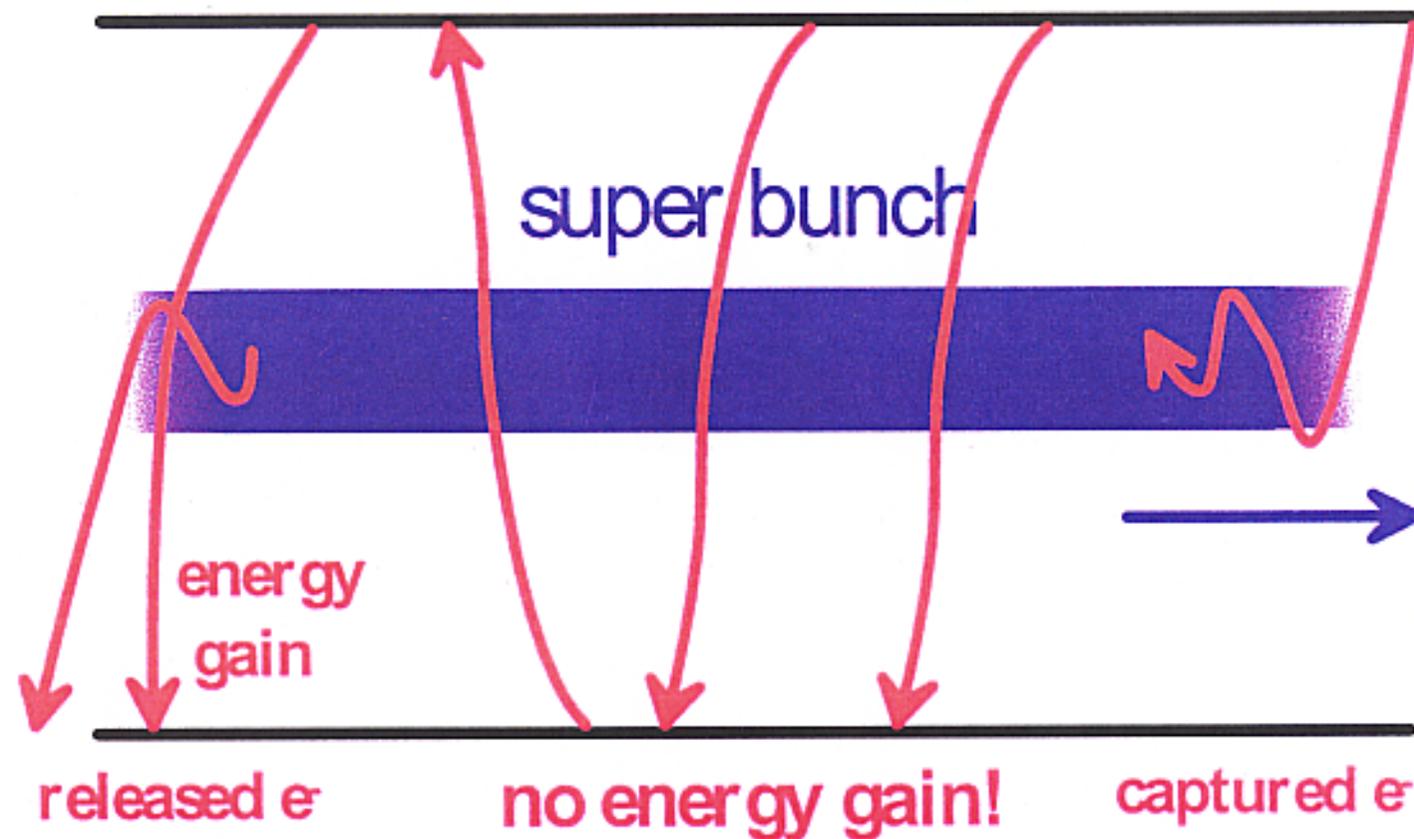
or, with two IPs,

$$|\kappa_-| = k |1 + \exp(i(\Delta\phi_x - \Delta\phi_y - \Delta Q\pi))| \quad \text{with}$$

$$k = \frac{n_{\text{par}} K_s \beta_s}{2\pi} = \frac{n_{\text{par}} N_b r_p}{\pi (\gamma \epsilon_y) n_{\text{sep}}^2}$$

where n_{sep} separation in σ , n_{par} no. of parasitic collisions around each IP, $\Delta\phi_{x,y}$ phase advance between the two IPs. For the LHC: $k \approx 0.005$. With 45° and 135° coupling is zero, for $\Delta\phi_x - \Delta\phi_y - \Delta Q\pi = n2\pi$. Another (better?) approach: choose $\Delta\phi_x \& \Delta\phi_y = n2\pi$.

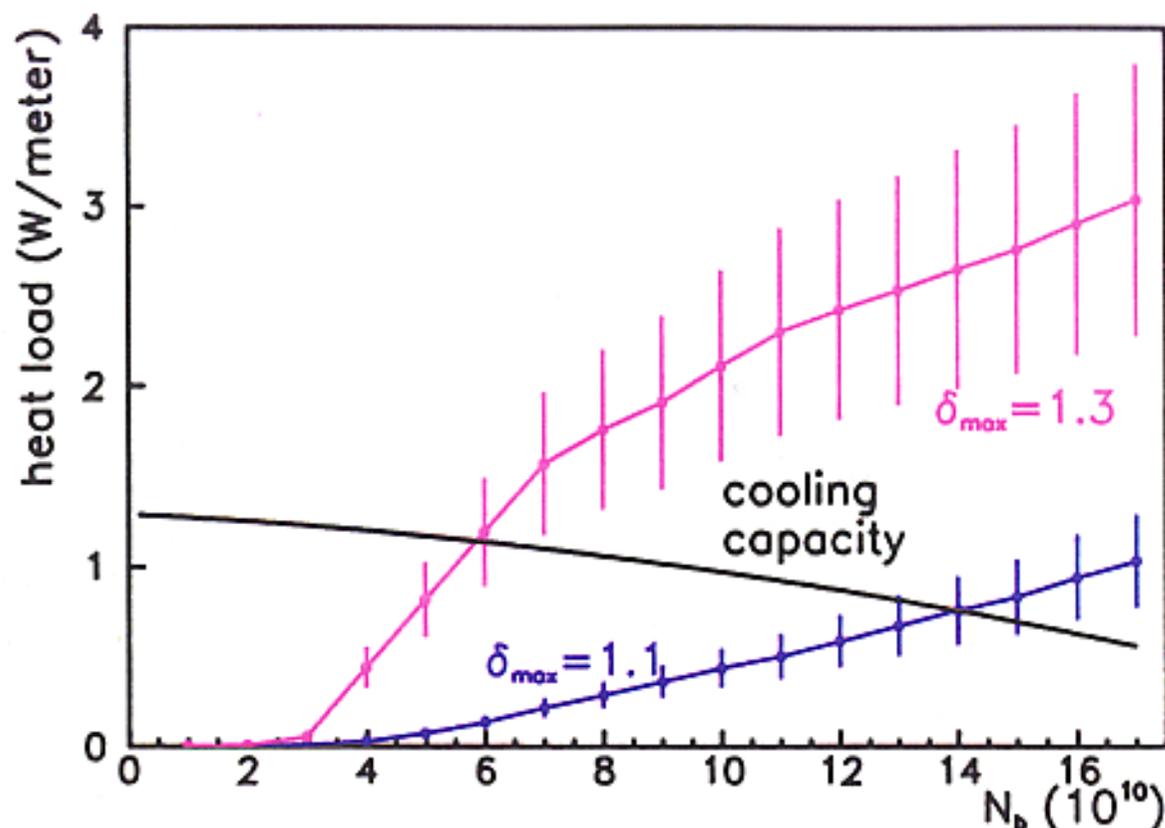
much less e^- multipacting for superbunch



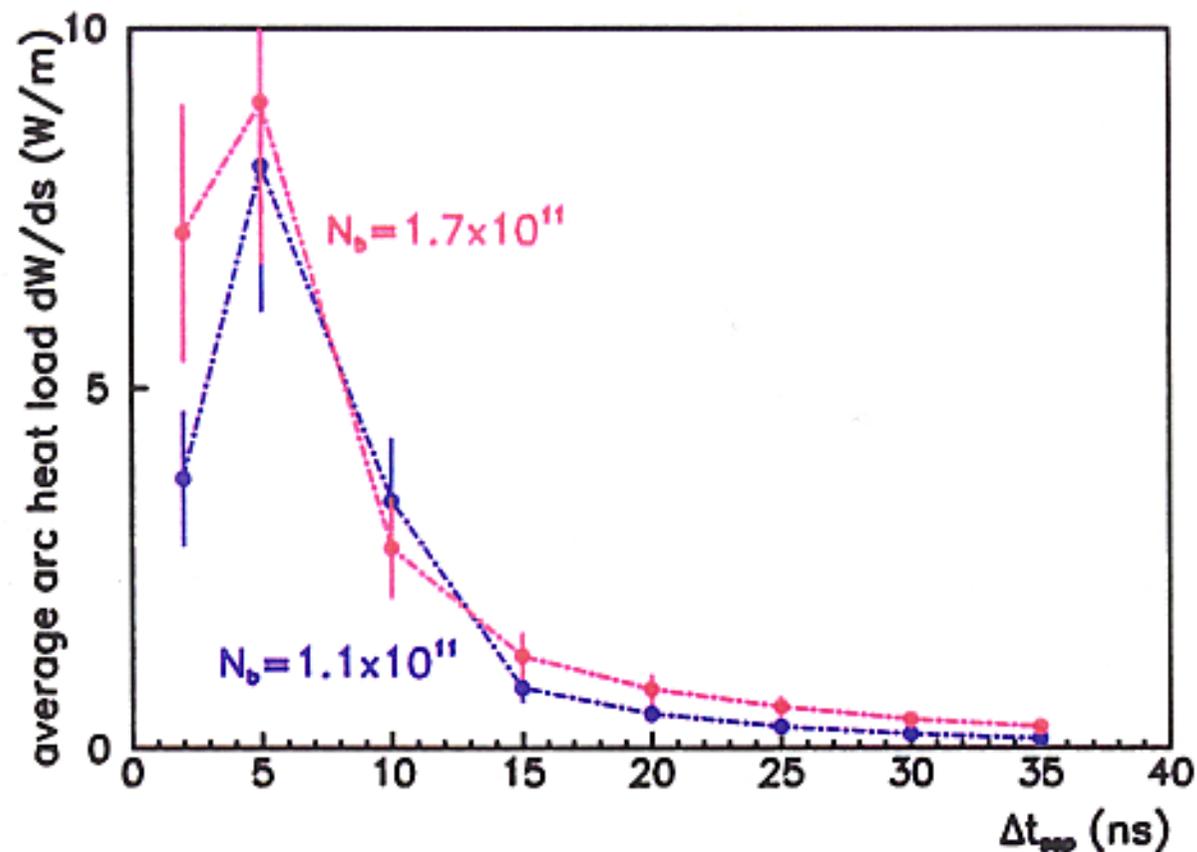
Schematic of **electron motion** during passage of **superbunch**; most electrons do not gain energy; beam profile important (schematic after J. Wei & R. Macek).

electrons incident on the chamber wall deposit
energy (typically a few 100 eV per e^-)

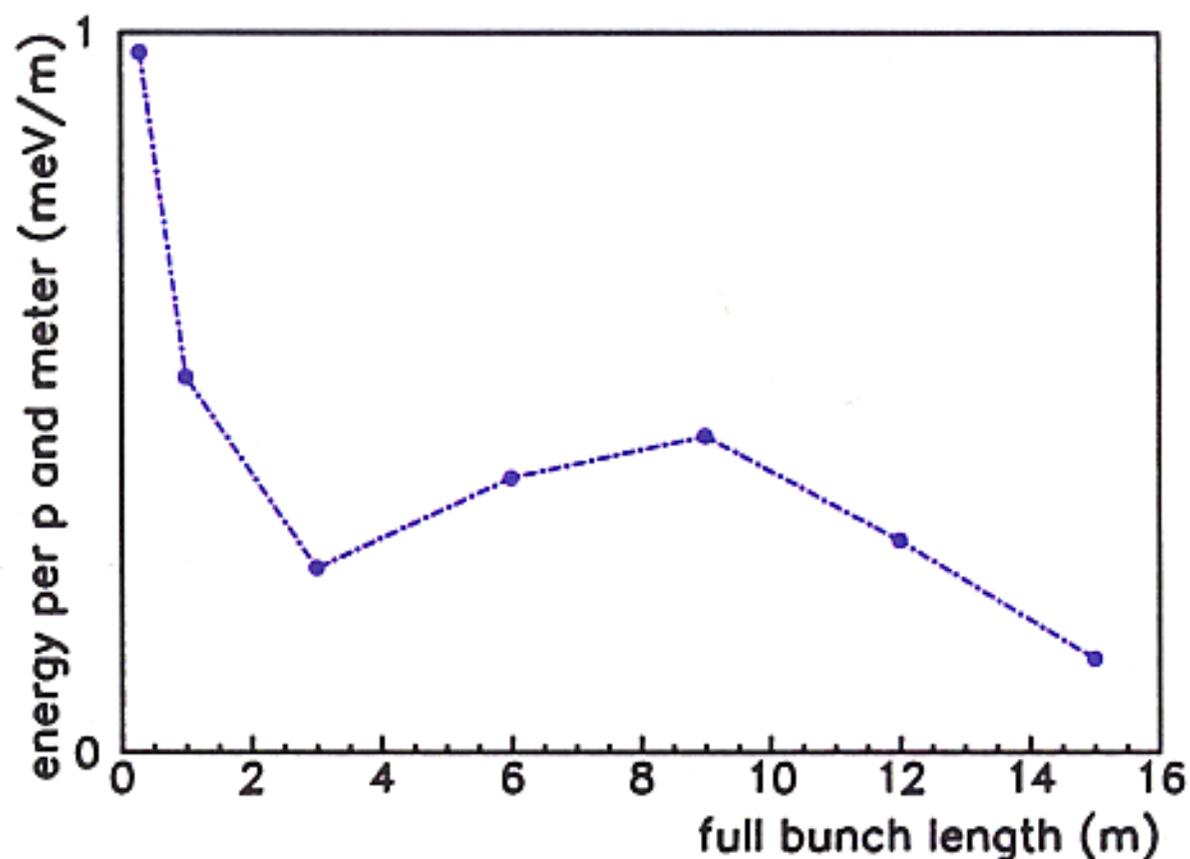
electron heat load can be much larger than heat
load from synchrotron radiation and is a major
concern for superconducting hadron rings



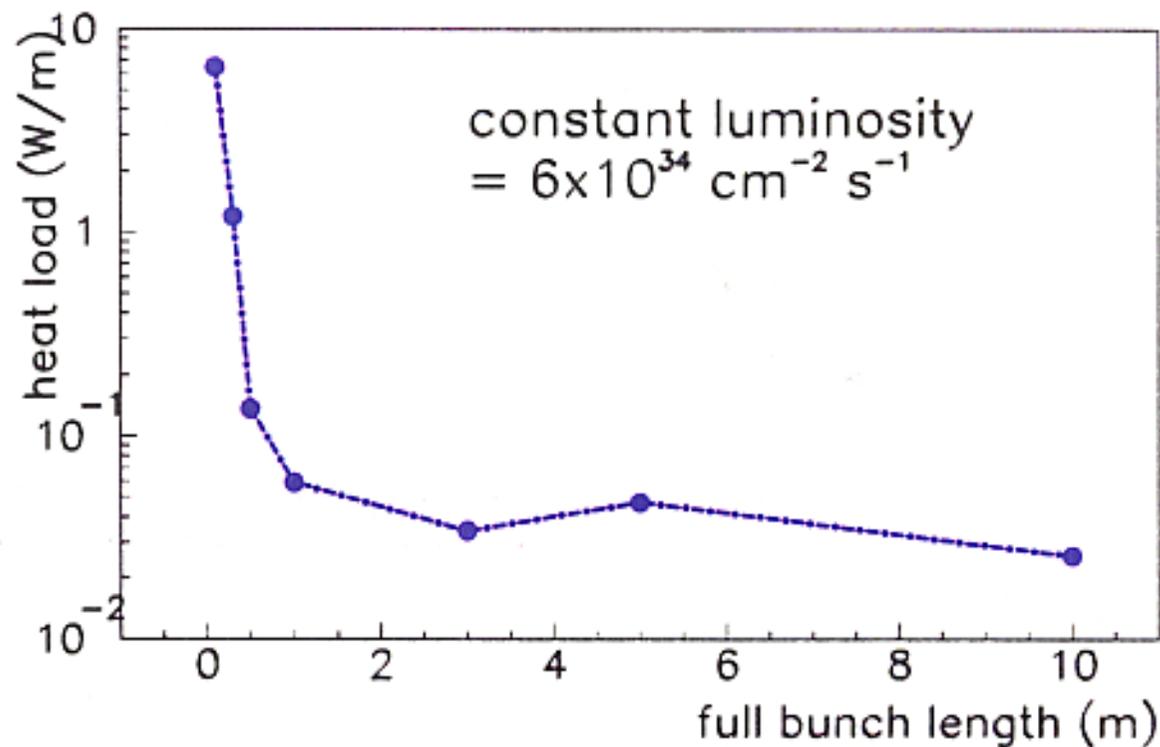
Simulated average LHC arc heat load and cooling capacity vs. bunch population N_b , for two values of δ_{max} . Other parameters are $\epsilon_{max} = 262$ eV, $R = 5\%$, $Y = 5\%$, and elastic electron reflection is included.



Simulated average **LHC arc heat load** vs. **bunch spacing**, for $\delta_{max} = 1.1$ and two bunch populations.



Simulated average **energy deposition per proton** vs. **full bunch length** for LHC dipole; line density $\lambda = 10^{12} \text{ m}^{-1}$ with 10% rising and falling edge.

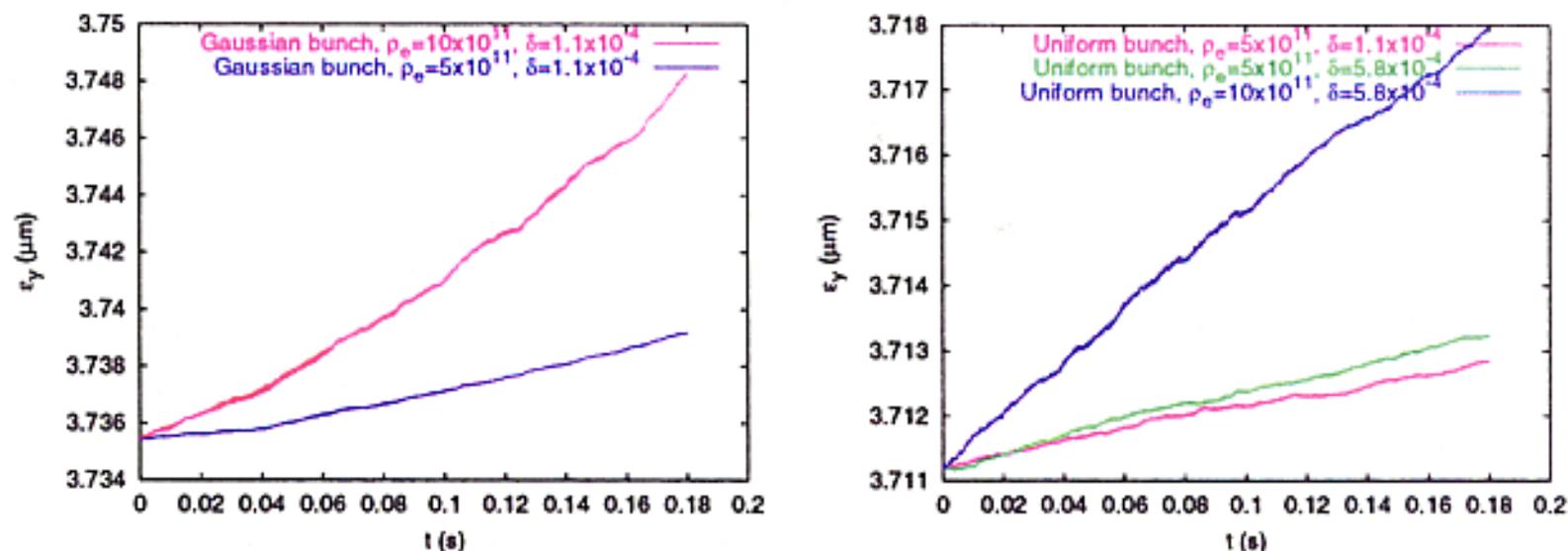


Simulated heat load in an LHC arc dipole due to the electron cloud vs. superbunch length for $\delta_{\max} = 1.4$, considering a constant flat top proton line density of $8 \times 10^{11} \text{ m}^{-1}$ with 10% linearly rising and falling edges. No. of bunches is varied to keep constant luminosity.

let us assume that e^- cloud builds up
what is the difference of Gaussian and flat
bunches?

question can be answered by simulation with
HEADTAIL code (G. Rumolo)

LHC e^- cloud instability for Gaussian and flat profiles



simulated norm. emittance growth for Gaussian (left) & flat bunches (right), 2 different e^- densities & rms momentum spreads; $l_{\text{flat}} = 4\sigma_z$ (G. Rumolo)

Energy Loss from Synchrotron Radiation

average energy loss per turn

$$U_0 = \frac{4\pi}{3} r_p m_p c^2 \frac{\gamma^4}{\rho}$$

for LHC, $\rho \approx 2700$ m, and $U_0 = 6710$ eV, or $\Delta\delta \approx 10^{-9}$.

average increase in variance of energy per turn

$$\Delta\delta^2 = \frac{55}{24\sqrt{3}} r_p \lambda_p \frac{\gamma^5}{\rho^2}$$

or $(\Delta\delta^2)^{1/2} \gamma m_p c^2 \approx 560$ eV for LHC

(momentum compaction $\alpha = 3.47 \times 10^{-4}$, circumference $C = 26.7$ km)

longitudinal equations of motion

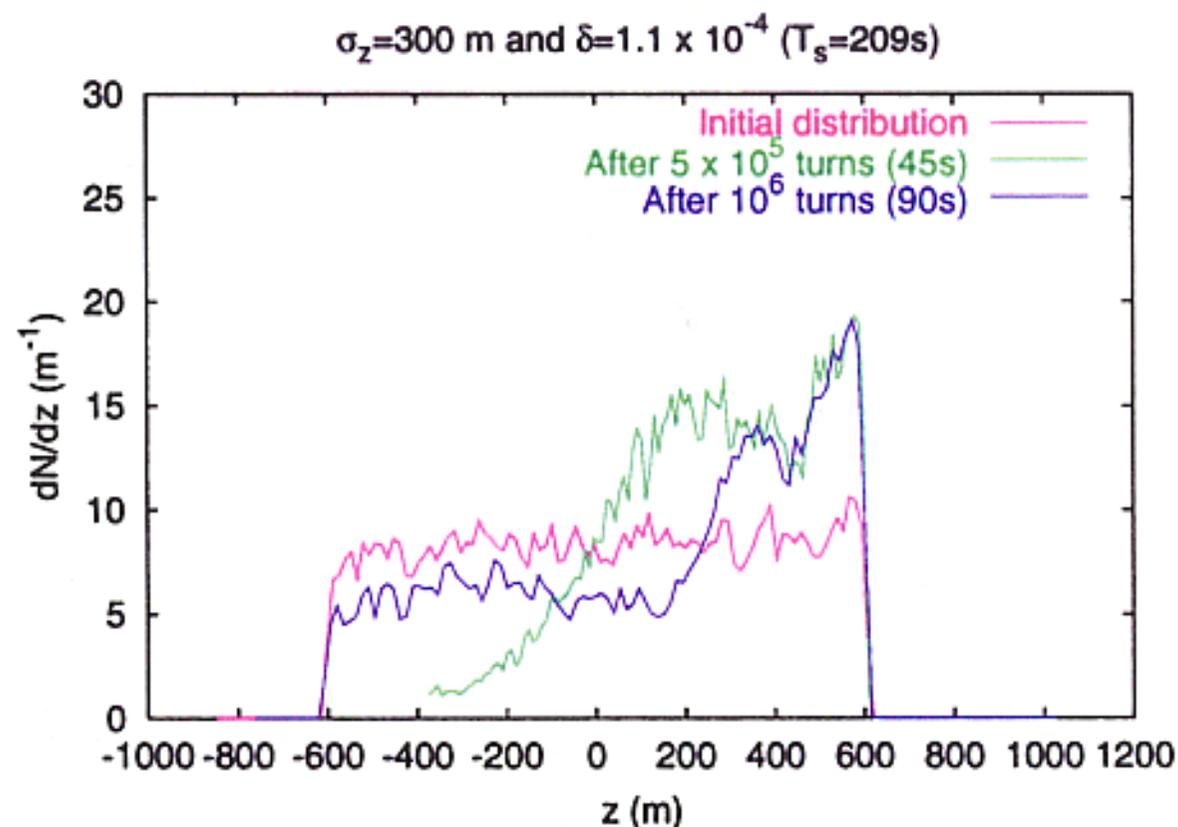
$$\frac{d\delta}{dt} = -\frac{\Delta\delta}{T_0} \quad \text{and} \quad \frac{ds}{dt} = \frac{\alpha C \delta}{T_0}$$

motion of particle with $\delta(0) = \delta_0$ stops if $\delta = 0$.

maximum traversed longitudinal distance:

$$s = \frac{\alpha C}{2\Delta\delta} \delta_0^2 \approx 56 \text{ m}$$

for $\delta_0 \approx 1.1 \times 10^{-4}$.



Profiles of 1200-m long superbunch in LHC at various times, with barrier bucket & synchrotron radiation; head is on the right; simulation by HEADTAIL code (G. Rumolo)

Radiation Damping

synchrotron radiation damping time the same as for normal bunches!

$$\frac{1}{\tau_{SR,\delta}} = \frac{1}{2T_0} \left(\frac{dU_{\text{rad}}}{dE} \right)_0$$

$$\tau_{SR,\delta} = \left(\frac{3(m_A c^2)^3}{2e^2 c^3 r_A Z^2} \right) \frac{1}{B^2 E} \left(\frac{C}{2\pi\rho} \right) \approx \frac{8322 \text{hr}}{E[\text{TeV}] B[\text{T}]^2} \left(\frac{C}{2\pi\rho} \right) \frac{A^4}{Z^4}$$

effective on much longer time scale

examples:

LHC: $\tau_{SR,\delta} \approx 26 \text{ hr}$

LHC-II at 14 TeV: $\tau_{SR,\delta} \approx 3.3 \text{ hr}$

Intrabeam Scattering

Theory of Bjorken & Mtingwa, Part.Accel.13:115-143,1983

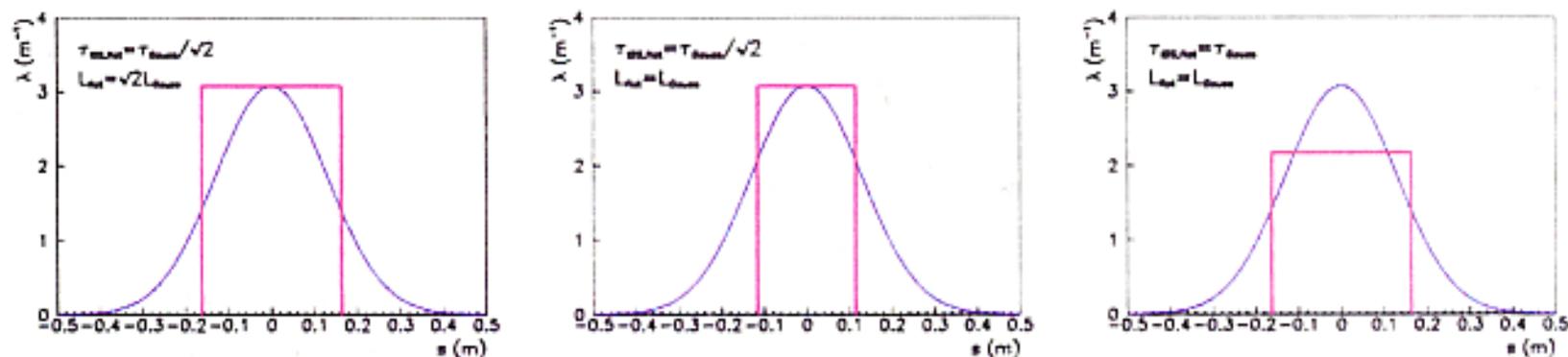
IBS for flat bunch or superbunch is the same as for coasting beam

(note: coefficient $\tilde{\Gamma} = \Gamma$ for bunched beam; $\tilde{\Gamma} = \Gamma/\sqrt{2}$ for unbunched beams; difference arises from integration over $\rho_1(s)\rho_2(s)$ for Gaussian or flat distributions!)

$$\frac{1}{\tau_{\text{IBS,flat}}} = \frac{2\sqrt{\pi}\sigma_z}{l_{\text{flat}}} \frac{N_{\text{flat}}}{N_{\text{Gaussian}}} \frac{1}{\tau_{\text{IBS,Gaussian}}}$$

$\frac{N_{\text{flat}}}{N_{\text{Gaussian}}}$	$\frac{l_{\text{flat}}}{\sqrt{2}\pi\sigma_z}$	$\frac{L_{\text{flat}}}{L_{\text{Gaussian}}}$	$\frac{1/\tau_{\text{IBS,flat}}}{1/\tau_{\text{IBS,Gaussian}}}$
1	1	$\sqrt{2}$	$\sqrt{2}$
$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$
$1/\sqrt{2}$	1	1	1

profiles for the 3 examples (incl. luminosity & τ_{IBS} ratios)



left: flat profile gives $\sqrt{2}$ times higher luminosity and IBS growth rates; right: flat profile gives equal luminosity and IBS growth rates.

Intrabeam Scattering Cont'd

absolute rates can be estimated using Karl Bane's approximation for high energy:

$$\frac{1}{\tau_p} = \frac{1}{\sigma_p} \frac{d\sigma_p}{dt} \approx \frac{\sqrt{\pi} r_p^2 c N(\log)}{8\gamma^3 \epsilon_{\perp}^{3/2} C \sigma_p^3} \frac{1}{\sqrt{\beta} \sqrt{\frac{1}{\sigma_p^2} + \frac{\langle \mathcal{H}_x \rangle}{\epsilon_x}}}$$

$$\frac{1}{\tau_x} = \frac{1}{\sigma_x} \frac{d\sigma_x}{dt} \approx \frac{\sqrt{\pi} \langle \mathcal{H}_x \rangle r_p^2 c N(\log)}{8\gamma^3 \epsilon_{\perp}^{5/2} C \sigma_p} \frac{1}{\sqrt{\beta} \sqrt{\frac{1}{\sigma_p^2} + \frac{\langle \mathcal{H}_x \rangle}{\epsilon_x}}}$$

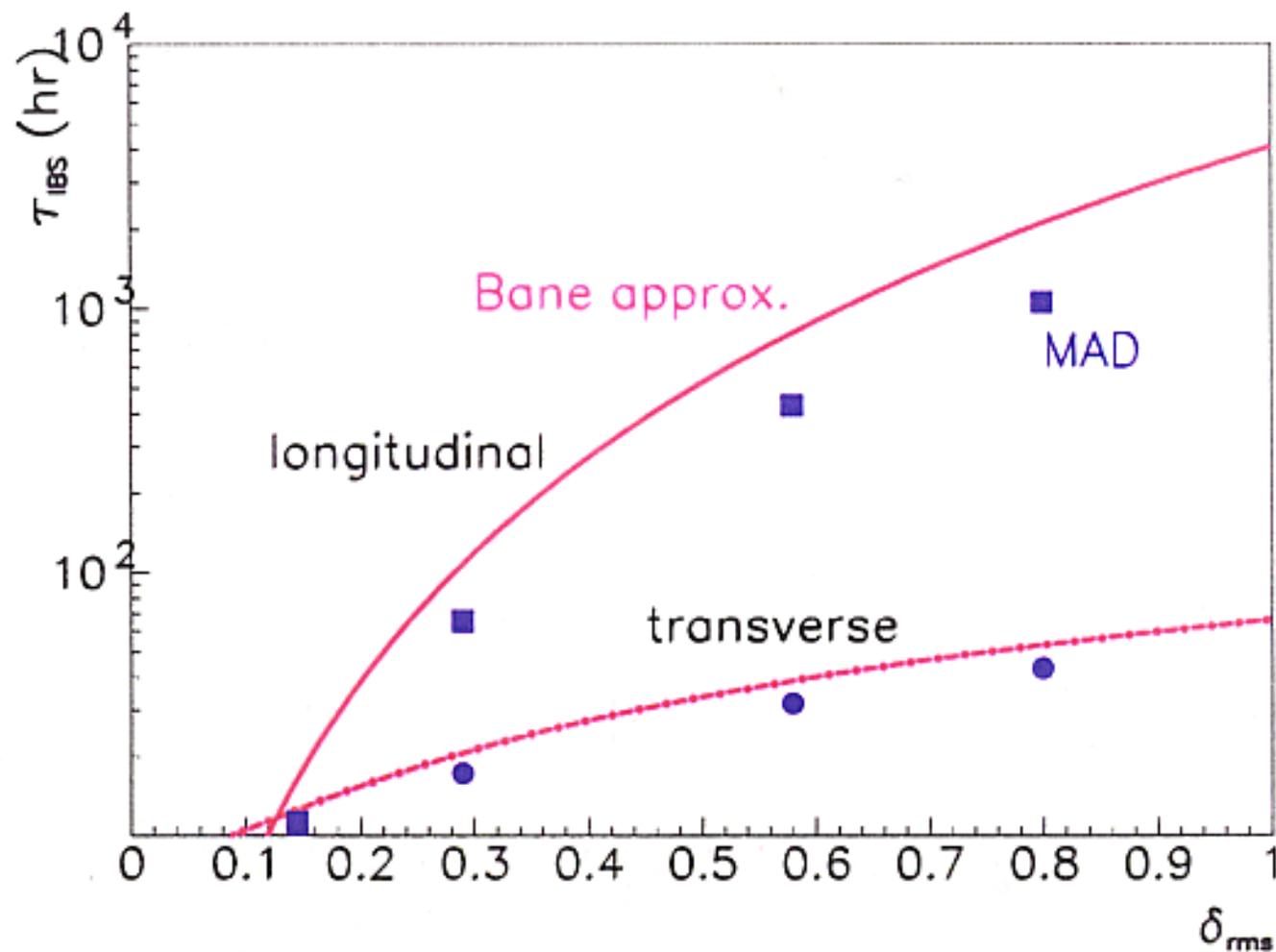
where we have assumed round beams;

use LHC parameters: $\langle \mathcal{H}_x \rangle \approx 0.03$ m, $C = 26.7$ km,

$\lambda_p = 2.1 \times 10^{12} \text{ m}^{-1}$ $\epsilon_x = \epsilon_y = 5 \times 10^{-10}$ m, $E = 7$ TeV, Coulomb

logarithm $(\log) \approx 24$.

comparison of MAD IBS calculation (Bjorken-Mtingwa)
and **Bane's approximation**: growth times vs. rms
momentum spread



Conclusions 1: hybrid crossing

advantages:

- local correction, *i.e.*, compensation of the long-range tune shift at each IP independently
- less sensitive to errors in the arcs
- allows for different crossing angles at the two IPs
- symmetrizes the two IPs (equal backgrounds in the two experiments!)
- smaller tune footprint
- larger diffusive aperture

disadvantages:

- introduces coupling through the arcs, which might complicate or degrade the operation

Conclusions 2: flat or super-bunches

- factor $\sqrt{2}$ higher luminosity
- for luminosity beam profile is important, not length
- intrabeam scattering also larger, unless energy spread is increased
- long superbunches avoid PACMAN effect

Conclusions 3: electron cloud, IBS, SR

- long superbunches suppress electron cloud build up and reduce heat load (details of longit. profile important)
- approximate IBS formula agrees with exact (MAD) calculation to within a factor of 2 over large parameter range; sensitivity to momentum spread
- w/o acceleration SR energy loss (or longitudinal wake) can deform the superbunch, if too long

11.4 RF parameters for LHC super-bunches

We discuss possible longitudinal RF parameters for a 300 m long super-bunch in the LHC with 1 A DC current. Assuming that the super-bunch is obtained by merging some 3000 LHC bunches with ultimate intensity, a longitudinal emittance larger than or equal to 15 keVs can be anticipated. This corresponds to an energy spread of $\pm 10^{-3}$ and requires a peak voltage of 3.4 MV for a sine-wave barrier bucket at 10 MHz or 680 kV for a harmonic RF system at 500 kHz. As shown in Table 26, proportionally higher voltages are required at higher RF frequencies. In case of barrier bucket, the super-bunch would have a smooth parabolic edge extending over about 20 ns. The necessary 500 kHz RF system could be made of 15 low Q /low impedance cavities, each one 1 m long with a diameter of 1.5 m and providing 45 kV. These could be septum cavities, in view of the limited beam separation.

RF frequency (single sine-wave)	100 MHz	40 MHz	10 MHz
peak voltage	34 MV	13.6 MV	3.4 MV
RF frequency (harmonics)	500 kHz ($h = 44$)	22 MHz ($h = 1780$)	165 MHz ($h = 13350$)
number of bunches	1	40	300
peak voltage	680 kV	27 MV	202 MV

Table 26: Parameters of barrier bucket (top) or harmonic RF systems (bottom) at different frequencies for an LHC super-bunch.