MUON COLLIDER: CONCEPTION AND SOME DISTINCTIVE FEATURES OF PHYSICS POTENTIAL

Feodor F. Tikhonin,* Institute of High Energy Physics, Protvino, 142284, Russia

Abstract

Machines with $\mu^+\mu^-$ colliding beams have a number of distinctive features compared to the $e^+e^-$ colliders. A few of them are briefly outlined.

1 CONCEPTION

Concept of a muon collider has been around for a while: [1] [2], [3], [4]. The main impetus for such an idea was the desire to get rid of the unpleasant synchrotron radiation, because of which so suffer the cycling electron-positron colliders. The muons being as much as $\approx 200$ times heavier than electrons undergo this disease to a much less extent because the intensity of synchrotron radiation is proportional to the mass$^{-4}$ of radiating particle. The interest of such a concept [3, 4] offers the possibility of making a high luminosity accelerator. From the other hand the physics potential of muon collider recently was enriched by possibility to build the "Higgs boson factory". We know what rich harvest was gathered from the "Z boson factories" at CERN (LEP-I) and at SLAC (SLC) and therefore such facility might provide a unique instrument for particles physics research.

2 PHYSICS MOTIVATION

Apparently one of the main objectives of present-days elementary particles physics are to find the Higgs boson and investigate its properties with as high precision as possible. It is hoped that this particle will be revealed at the forthcoming LHC machine. However there exists the so called Bjorken process having muons beyond this region, more exactly, if the Higgs mass

\[ m_H \]

is the energy squared of collision. Also $m_\mu$, $M_H$ and $M_W$ are the masses of the $\mu$, $Z$ meson, Higgs scalar and W boson, respectively, and $\Gamma_H$ is the decay width of Higgs. For the fine structure constant, $\alpha$, we use an approximate value 1/128 and for the sine of electroweak mixing angle the value of $\sin^2\theta_W \approx 0.23$. As can be seen from this equation, the cross section due to the Higgs boson exchange alone reaches its maximum, $\sigma_{\mu\mu\to WW} = 0.067\text{pb}$ at the c.m. energy $\sqrt{s} \approx 2m_Z$ while the "conventional" cross-section (due to $\gamma, Z^0$ and $\nu$ exchange) reaches at this point the value of $\approx 15$ pb. Therefore the "Higgs boson factory" is feasible only in the region $m_H < 2m_W$. Partly in view of this it is expedient to search for other processes, where the interaction of Higgs within the lepton sector would be involved. In this talk we represent two examples of such processes, $\mu^+\mu^- \to H Z^0$ and $\mu^+\mu^- \to H \gamma$, and will consider them as complementary to that of resonant Higgs scalar production. Note, that second of the processes above is negligibly small at the tree level in the case of electron-positron collisions, justifying word "distinctive" in the title of present talk.

3 ASSOCIATED $H Z^0$ PRODUCTION [5]

We begin with the so called Bjorken process having muons as the initial state particles, $\mu^+\mu^- \to H Z^0$.

Usually in the course of cross-section calculations one uses the the s-channel diagram alone. Let us do it we but in addition take into account masses of initial muons. Then we obtain the following asymptotics of this process at $\sqrt{s} \to \infty$:

\[ \sigma_{\mu^+\mu^-\to HZ^0} \propto \frac{\alpha^2 g_Z^2}{\sin^2(2\theta_W)} \frac{m_H^2}{m_Z^2} \]

It can be seen that despite the fact that this diagram is pure $s$–channel one, the corresponding cross section is not falling with energy, but approaches a constant limit, whose value is equal to $\approx 1.2 \cdot 10^{-2} \text{fb}$. Concerning the angular dependence of this cross section, it could be seen, that this distribution is flat, indicating that it comes entirely from the $J = 0$ partial wave. Obviously, this behavior contradicts unitarity condition, which requires $\sigma_{J=0} \leq s^{-1}$ at high energies. Now we calculate the contributions to this cross section, stemming from the cross channels diagrams,
t- and u- one. It turns out that corresponding contribution is again equal exactly to the value of $\simeq 1.2 \cdot 10^{-2} fb$. Corresponding angular distribution is also flat. At last, let us take into account interference term between the two classes of the diagrams above. We found that it is equal exactly to $\simeq -2.4 \cdot 10^{-2} fb$. Adding all three contributions we obtain result, which removes a seeming contradiction. As it must be, the asymptotic form of cross section for the process under consideration at $\sqrt{s} \to \infty$ acquires "desired" form, i.e. it falls with energy.

$$\sigma(\mu^+\mu^- \to HZ^0)_{\text{asympt}} = \frac{1}{3} \frac{\pi \alpha^2}{\sin^4(2\theta_{EW})} \frac{1}{s}$$

(3)

The cancellation obtained reflects the most fundamental property of electroweak theory. This is a consequence of unitarity condition at tree level which must be fulfilled in any non-Abelian gauge theory with the symmetry broken in a manner like the Higgs mechanism [6].

In order to extract information about the Higgs - lepton sector interplay let us look once more at the individual contributions to the discussed cross section. In this respect it is worthwhile to note that all three contributions reach their high energy asymptotic (constant) values not simultaneously. Those stemming from the sum of the t-channel and u-channel go to the plateau at the energy around 1–2 TeV. The negative contribution reaches its minimum value at $\sqrt{s} \sim 2.5$ TeV, while the part of cross section, corresponding to the s-channel becomes constant (at nonnegligible muon mass) far away from 1–2 TeV region. However, in spite of different characters of the behaviour of these contributions, it seems unlikely that this difference will be observed experimentally owing to the small value of the $\mu$-meson mass. Partly because of that, in the next paragraph we will consider the process to which the Higgs-Gauge-Boson vertex is not involved.

4 ASSOCIATED $H\gamma$ PRODUCTION [5]

We now turn to process analogous to those just described, but which is free from the s-channel diagram complication. The differential cross section of the process $\mu^+\mu^- \to H\gamma$ for the case when photon is hitting a non-forward detector, and when we neglect the muon mass (apart from where it affects the muon - Higgs boson coupling constant), reads as follows 1:

$$\frac{d\sigma(H\gamma)}{d\cos \vartheta_\gamma} = \frac{\pi \alpha^2}{2 \sin^2 \theta_{EW}} \frac{m^2_H}{m^2_H} \frac{1}{s^2 - m^2_H} \frac{1}{1 - \cos^2 \vartheta_\gamma},$$

$$\frac{m^2_H}{m^2_H} \ll 1 - |\cos \vartheta_\gamma|,$$

(4)

where $\vartheta_\gamma$ is the angle emission of the photon with respect to the beam of initial $\mu^-$. Subscript "zero" means that the cross section has been calculated at tree level. With the yearly integrated luminosity of $L \simeq 10^3$ fb$^{-1}$ expected at future $\mu^+\mu^-$ colliders, one could collect 20 to 30 $H\gamma$ events (detector efficiency is supposed equal to 1, and acceptance $\approx 1$). The signal which mainly consists of a photon and $b\bar{b}$ pairs in the low Higgs mass range or $WW/Z\gamma Z^0$ pairs for Higgs masses larger than ~ 200 GeV, is extremely clean. The background should be very small since the photon must be very energetic and the $b\bar{b}$ or $WW/Z\gamma Z^0$ pairs should peak at an invariant mass $m_H$. Therefore, despite the low rates, a clean signal gives a good possibility to detect these events.

As can be seen from the Eq.(4) the tree - level cross section is singular $\sim \sqrt{s} \to m_H$. In order to overcome this difficulty we need to calculate the Quantum Electrodynamics (QED) corrections to the tree-level amplitude. That is the aim of the next paragraph.

5 RADIATIVE CORRECTIONS TO THE PROCESS $\mu^+\mu^- \to H\gamma$

Due of the lack of the space here only very concise sketch of RC will be given. For the comprehensive information I refer to [7]. For the process under study 1) the first order leading logarithmic correction were obtained and 2) the contribution of the higher order perturbation theory were taken into account in the leading logarithmic approximation. It is expedient to give here expression for the case 2). The master formula for radiative corrections has the form of the Drell–Yan cross section. So, we suggest to write the result as a convolution of the lepton structure functions with the shifted cross section of the hard subprocess. It reads

$$\frac{d\sigma(z, L)}{d\cos \vartheta_\gamma} = \frac{1}{2} \frac{\beta(1 - z)}{\beta(1 + z)\alpha (\beta^2)},$$

$$\beta = \frac{2\alpha}{\pi} \ln \frac{m_H}{m_H},$$

(5)

and the lower limits for integration over $z_{1,2}$ are as follows:

$$z_{1,2}^{\min} = \frac{m^2_H + \sqrt{s}\omega_{th}(1 + \cos \vartheta_\gamma)}{s - \sqrt{s}\omega_{th}(1 - \cos \vartheta_\gamma)},$$

(6)

where $\omega_{th}$ is the experimental energy threshold of the photon registration. D–function has the following form:

$$D(z, L) = \frac{1}{2} \beta(1 - z)\alpha^2 \frac{1}{1 + z^2}(1 + O(\beta^2)),$$

$$\beta = \frac{2\alpha}{\pi} \ln \frac{m_H}{m_H} - 1,$$

(7)

1 Another factor, $1/(1 - \cos^2 \vartheta_\gamma)$, is seemingly singular but it is not dangerous when masses of initial muons are taken into account.
One can easily see by direct inspection that the expression of Eq.(5) is smooth. It achieves its maximal values in the vicinity of the point \( \sqrt{s} \sim m_H \) and falls rapidly to zero as \( \sqrt{s} \to m_H \). Cross section slowly falls with increasing \( \sqrt{s} \).

The associated \( H \gamma \) pair can be produced through loop diagrams of massive particles (gauge bosons, heavy quarks) also [8]. In this case the cross section behaviour is quite different from that of tree-level with QED corrections just considered. Contrary to this last case the loop induced process has the negligible cross section at the threshold and slowly rises as the energy grows. Both cross section values coincide at the point \( \sqrt{s}/2 \sim m_H \). Before this point the tree-level QED corrected process dominates while after this point the loop induced process dominates. So, this process delivers the unique possibility of studying simultaneously the \( (\mu \bar{\mu} H) \) vertex (in the region of \( \sim m_H \leq \sqrt{s} < 2m_H \)) and, for instance, \( (t \bar{t} H) \) or \( (W^+ W^- H) \) vertices (in the region of \( \sqrt{s} > 2m_H \)).

6 CONCLUSION

Muon colliders will offer an excellent opportunity for the particle physics investigations.

7 REFERENCES


