# An analysis of the coherent synchrotron radiation effect in an energy-recovery linac by first-order transfer matrix

R. Hajima*
Japan Atomic Energy Research Institute, Tokai, Ibaraki 319–1195 Japan

*Abstract*

We propose a novel approach to the analysis of the coherent synchrotron radiation (CSR) effect on beam dynamics in energy-recovery linacs (ERL). Since an ERL return-arc consists of a large number of dipole and quadrupole magnets, it is not a practical way to search an optimum design by time-consuming particle simulations. We show that a first-order transfer matrix can be applied to calculate the emittance growth due to the CSR in the linear regime. The optimum design of ERL arcs such as triple-bend achromat lattice is easily obtained by matrix calculation. As an example, optimization of a 3GeV ERL arc is presented.

## INTRODUCTION

Dilution of electron beam emittance caused by coherent synchrotron radiation (CSR) at a bending path has been a critical problem in the design of bunch compressors for X-ray free-electron lasers (XFEL) and return arcs for energy-recovery linacs (ERL) since the problem was pointed out by Derbenev and Carlsten[1]. Many studies have been devoted to this problem by experimental[2], analytical[3], and numerical approach[4] .

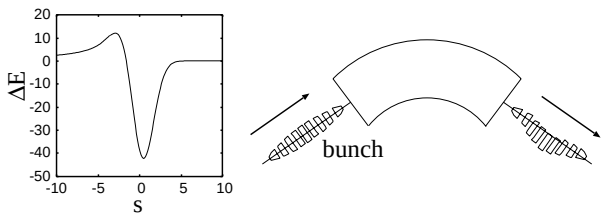


Figure 1: Energy modulation induced by CSR wake potential results in emittance dilution at the bend exit

Figure 1 is a schematic view to show the emittance dilution by CSR. When an electron bunch travels in a bending path, CSR wake potential induces energy modulation in the bunch. Since the energy modulation for each electron depends on its longitudinal position in the bunch, displacement of electrons appears at the bending exit, which is the dilution of beam emittance. It should be noted that the longitudinal CSR wake depends little on the transverse position of electrons, and the emittance of each beamlet, that is slice emittance, is reserved, while the projection emittance is diluted by CSR. In the ERL light sources, the property of synchrotron radiation and undulator radiation is a strong function of the projection emittance, thus, the dilution of the projection emittance is a critical phenomenon.

Analyses of the CSR-induced emittance dilution have been conducted by numerical simulation. Several particle tracking codes have been developed for this purpose and contributed to the design of bunch compressors for SASE-FELs [4]. Since these simulation codes require long calculation time, they are not suitable to search an optimum achromatic lattice of an ERL-loop by scanning a number of parameters such as angle and radius of dipoles, position and strength of quadrupoles, sextupoles, beam envelope matching to straight sections. We, therefore, propose an alternative method for the CSR analysis based on a matrix calculation, which provides a simple method to scan large number of parameters and to optimize an ERL-loop[5].

## LINEAR ANALYSIS BY TRANSFER MATRIX

A first-order equation of electron motion in a uniform field of a dipole magnet is

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \delta_{csr} + \kappa[s - s_0]\right) \ , \qquad (1)$$

where the variables are $x$ the deviation of an electron from the ideal path in the bending plane, $x''$ second-order derivative of $x$ with respect to a coordinate along the ideal path $s$, $\rho$ the bending radius, and $\delta_0$ the initial momentum deviation at $s = 0$ normalized by the reference momentum. The last two terms on the right-hand side have been added to calculate the CSR effect: $\delta_{csr}$ the normalized momentum deviation caused by CSR in the upstream path ($0 < s < s_0$), $s = s_0$ the entrance of the bending magnet, $\kappa = W/E_0$ the normalized CSR wake potential in the bending path defined by CSR wake potential $W$ and the reference energy $E_0$.

We assume in the following discussion that (1) all the bending magnet have the same bending radius, (2) an electron bunch keeps constant longitudinal profile, (3) transient CSR effect at the entrance and exit of magnets is not large. These assumptions are valid for high-energy ERL light sources.

Under the above assumptions we can attribute the normalized CSR wake potential to individual electrons, and the electron motion is uniquely specified by a vector

$$\vec{x}(s_0) = (x, x', \delta_0, \delta_{CSR}, \kappa)^T \ . \qquad (2)$$

Tracking this vector according to Eq.(1) can be made by using first-order matrices.

* hajima@popsvr.tokai.jaeri.go.jp

Given an initial vector at the entrance of a bending magnet, $s = s_0$, we can solve the electron trajectory through the bending magnet and find a vector at the bending exit, $s = s_1$:

$$\vec{x}(s_1) = R_{bend}\ \vec{x}(s_0) \tag{3}$$

The transfer matrix of a sector bending magnet is derived by Green's function method[6]:

$$R_{bend} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

This is an extension of a well-known 3x3 R-matrix used in beam transport analyses with momentum dispersion. Similar matrices are obtained for a quadrupole and a drift, and the electron motion along a beam transport system can be calculated from these R-matrices.

The deviation of electrons from the design trajectory due to initial momentum error is described by momentum dispersion function:

$$\begin{aligned} &(\eta_x(s_1)\ \ \eta'_x(s_1)\ \ 1\ \ 0\ \ 0)^T \\ &\quad = \quad R_{0\to 1}\ \ (\eta_x(s_0)\ \ \eta'_x(s_0)\ \ 1\ \ 0\ \ 0)^T\ . \end{aligned} \tag{5}$$

The deviation of each electron in the $(x, x')$ phase space due to the initial momentum error is expressed by $\vec{d} = (\delta_0\eta_x,\ \delta_0\eta'_x)$.

Following this style, we introduce a CSR wake dispersion function, $\zeta_x(s)$, to track off-axis motion due to the CSR effect:

$$\begin{aligned} &(\zeta_x(s_1)\ \ \zeta'_x(s_1)\ \ 0\ \ L_b(s_1)\ \ 1)^T \\ &\quad = \quad R_{0\to 1}\ \ (\zeta_x(s_0)\ \ \zeta'_x(s_0)\ \ 0\ \ L_b(s_0)\ \ 1)^T\ , \end{aligned} \tag{6}$$

where $L_b(s_1)$ is the total bending path length for $0 < s < s_1$, and has a relation $\delta_{CSR} = \kappa L_b$. The deviation of each electron due to the CSR effect is expressed by $\vec{d} = (\kappa\zeta_x,\ \kappa\zeta'_x)$.

Energy spread caused by CSR is calculated for a Gaussian bunch:

$$\Delta E_{rms} \simeq 0.22\frac{eQL_b}{4\pi\epsilon_0\rho^{2/3}\sigma_s^{4/3}}\ , \tag{7}$$

where $Q$ is bunch charge, $\sigma_s$ is rms bunch length. This energy spread results in the deviation of bunch slices in the $(x, x')$ phase space after a bending path:

$$(D, D') = (\Delta\kappa_{rms}\zeta_x,\ \Delta\kappa_{rms}\zeta'_x)\ . \tag{8}$$

$$\Delta\kappa_{rms} = \Delta E_{rms}/E_0/L_b\ . \tag{9}$$

Unnormalized emittance at an arbitrary position along a beam transport is obtained by

$$\varepsilon^2 = (\varepsilon_0\beta_x + D^2)(\varepsilon_0\gamma_x + D'^2) - (\varepsilon_0\alpha_x - DD')^2\ , \tag{10}$$

where $(\alpha_x, \beta_x, \gamma_x)$ are Courant-Snyder parameters, $\varepsilon_0$ is the initial emittance. Terms related to the initial momentum error have been dropped from eq.(10), because the emittance is usually evaluated where the momentum dispersion is zero, $\eta = \eta' = 0$.

## EMITTANCE COMPENSATION IN THE LINEAR REGIME

The linear analysis derived in the previous section gives good approximation of electrons motion in high-energy ERLs, where $x/\rho \ll 1$ and $\delta \ll 1$. In this linear regime, all the bunch slices align on a single line at the exit of achromatic cell including bending paths as shown in Fig.2, and 3. These figures indicate that the projection emittance depends on the orientation of the CSR kick and the phase ellipse, and has a minimum value, when the achromatic cell is designed so that the CSR kick coincides with the orientation of the phase ellipse at the cell exit. We show that an achromatic cell for such minimum emittance can be designed by matrix calculation.

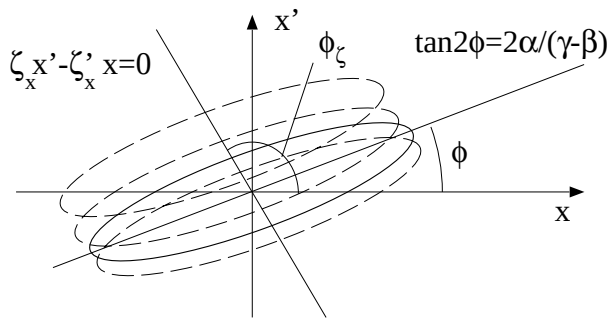


Figure 2: Two-dimensional phase ellipse in the $(x, x')$ plane, and displacement of beam slices due to the CSR kick. $(\alpha, \beta, \gamma)$ are Courant-Snyder parameters.

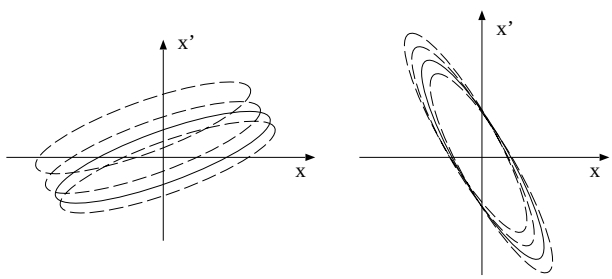


Figure 3: The CSR-induced emittance dilution becomes minimum, when the CSR kick coincides with the phase ellipse orientation.

We consider CSR-induced emittance growth through a triple-bend achromatic cell (TBA) as a part of an ERL-loop (Fig. 4).

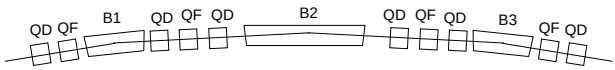


Figure 4: A triple bend acromatic cell: $\rho = 25$m, $\theta = 3 + 6 + 3 = 12$degree.

We can use the matrix method to search an optimum configuration by scanning cell parameters. In the design, we keep isochronous condition, $R_{56} = 0$. Figure 5 shows betatron and dispersion functions for a TBA cell, which gives minimum emittance dilution. It shows that $\beta_x$ and $\zeta_x$ have a similar envelope after the last bending magnet, which corresponds to the coincidence between the CSR kick and the beam ellipse orientation in the $(x, x')$ phase space.

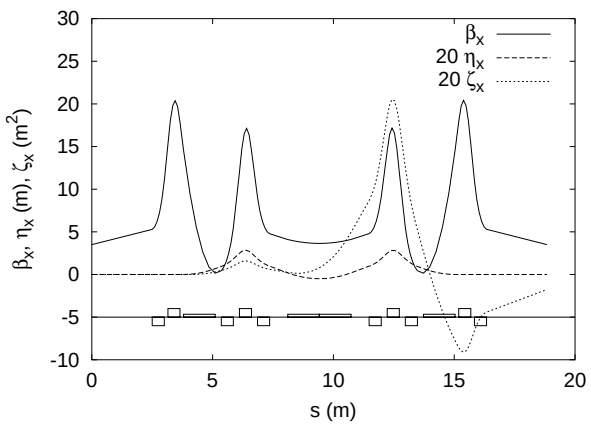


Figure 5: Betatron function $\beta_x$, momentum dispersion $\eta_x$ and CSR wake dispersion $\zeta_x$ of an optimized triple-bend achromat. The envelope of the CSR wake dispersion matches the betatron envelope after the last bending magnet.

The emittance compensation in the optimized TBA cell has been confirmed by particle tracking code `ELEGANT` including CSR calculation.[7] We assume the initial electron bunch parameters as follows: central energy 3.07GeV, bunch charge $Q = 770$pC, normalized rms emittance $\varepsilon_{n,0} = 0.1$mm-mrad, longitudinal size $\sigma_s = 30\mu$m (100fs), uncorrelated energy spread $\sigma_E/E_0 = 0.02\%$, and Gaussian distribution of electrons in the 6D phase space. Projection emittance at the cell exit is calculated with scanning Courant-Snyder parameters at the cell entrance: $-5.0 < \alpha_x < 3.0$, $\gamma_x = 0.29m^{-1}$.

Figure 6 shows the calculated projection emittance as a function of $\alpha_x$ at the third bend exit. It has been confirmed that the projection emittance is a function of the beam envelope inside the TBA cell, and the CSR-induced emittance dilution can be estimated precisely by matrix method. We also plotted simulation results including the transient CSR effect. It is found that the matrix approach gives an optimum design for the emittance compensation even when the transient CSR is included.

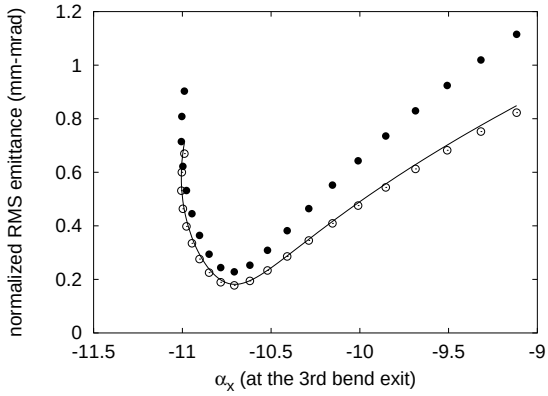


Figure 6: Emittance growth as a function of $\alpha_x$ at the 3rd bend exit. 1st-order matrix (solid line), particle tracking without transient CSR (○), particle tracking with transient CSR (●).

## CONCLUSION

We have shown an analysis of emittance dilution due to the CSR effect by using a first-order transform matrix method. Since the matrix approach enables us to scan a number of parameters quickly, it is widely applicable to the design optimization of achromatic cells for ERL light sources. We have seen that the CSR-induced emittance growth in an achromatic cell of a 3GeV ERL loop can be minimized by choosing an appropriate beam envelope inside the cell. This optimum cell design is easily obtained by matrix calculation.

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