Modulation of Trapped Ion Density Due to Passage of Bunch Train in KEK-PF Electron Storage Ring

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Abstract

We tried a theoretical model that gives modulation of trapped ion density.

- The residual gas ions are affected by periodic force due to passage of the beam and oscillate around beam orbit.
- The ion-cloud around the beam changes its size due to the oscillation of the ions.
- The theory of betatron oscillation in circular accelerators was applied to the motion of the trapped ions.

Bunch-by-bunch tune measurement in KEK-PF supported the theoretical prediction.

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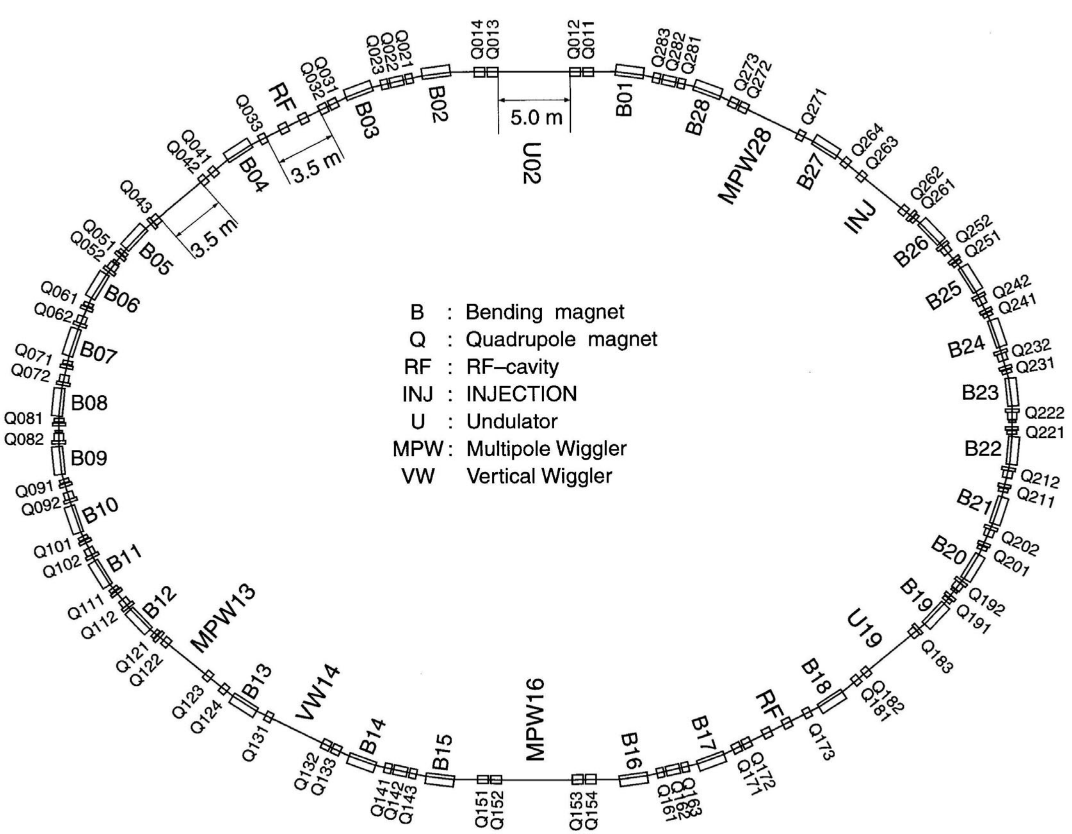
1. Introduction

KEK Photon Factory (KEK-PF)



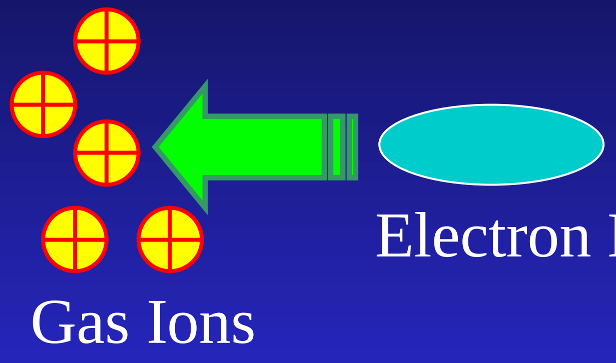
Circumference	C	187m
Energy	E	2.5GeV
RF Frequency	f_{RF}	500.1MHz
Bunch Spacing	t_{RF}	2 ns
Harmonic Number	h	312
Revolution Frequency	f_{rev}	1.6 MHz
Revolution Time	t_{rev}	624 ns
Total Current (Multi-Bunch)	I_{tot}	450 mA

Filling pattern in multi-bunch operation …

280 Bunches (1 Bunch Train) + 32 Empty Buckets (1 Bunch Gap)

2. Motion of the Ion Affected by Passage of the Bunch Train

Vertical motion of the ion affected by the passage of one bunch :



$$\begin{pmatrix} y_i(t+t_{RF}) \\ \dot{y}_i(t+t_{RF}) \end{pmatrix} = \underline{\begin{pmatrix} 1 & t_{RF} \\ 0 & 1 \end{pmatrix}} \underline{\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}} \begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}$$

$$= \underline{D}\underline{B} \begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}$$

t_{RF} ... Bunch spacing time

(2 ns in KEK - PF)

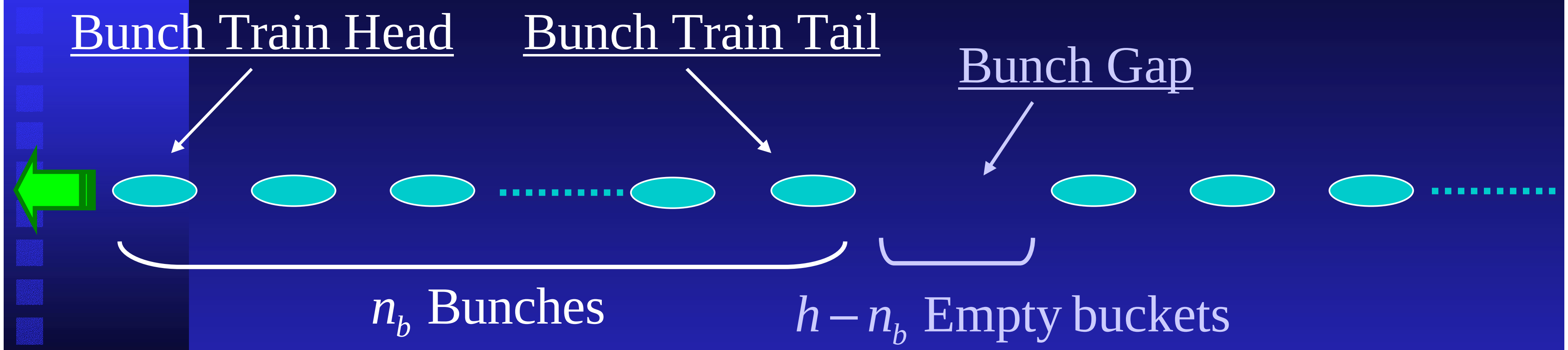
a ...Impulse due to the pasage of 1 bunch :

$$a = -\frac{2 r_p c N_b}{A\sigma_y(\sigma_x + \sigma_y)}$$

$\sigma_{x,y}$ ... Hor./Ver. Beam size

N_b ... Number of the electron in the bunch

Matrix for Passage of the Whole Bunch Train



Bunch Train Head

Bunch Train Tail

Bunch Gap

n_b Bunches

$h - n_b$ Empty buckets

$$\begin{pmatrix} y_i(t+t_{rev}) \\ \dot{y}_i(t+t_{rev}) \end{pmatrix} = D^{h-n_b}(DB)^{n_b} \begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}$$

$$= M \begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}$$

The stability condition of the ions can be written as

$$\frac{1}{2}\left|\mathrm{Tr.}\,M\right| \le 1$$

The trace depends on :

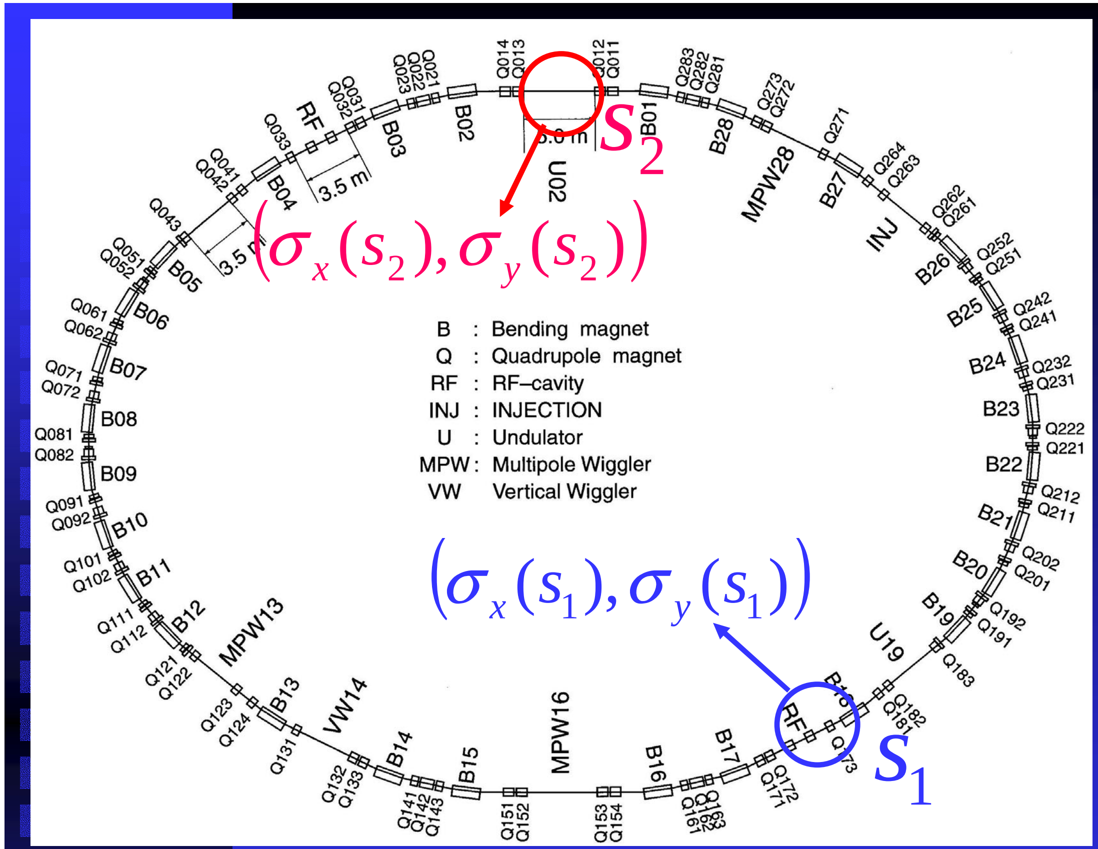
1) the configuration of the bunch train.

2) the beam size where the ions are trapped in the ring.

The stability condition differs in:

1) different configuration of the train.

2) different positions in the ring.



In General :

$$\mathrm{Tr.}\, M(s_1) \neq \mathrm{Tr.}\, M(s_2)$$

Because .....

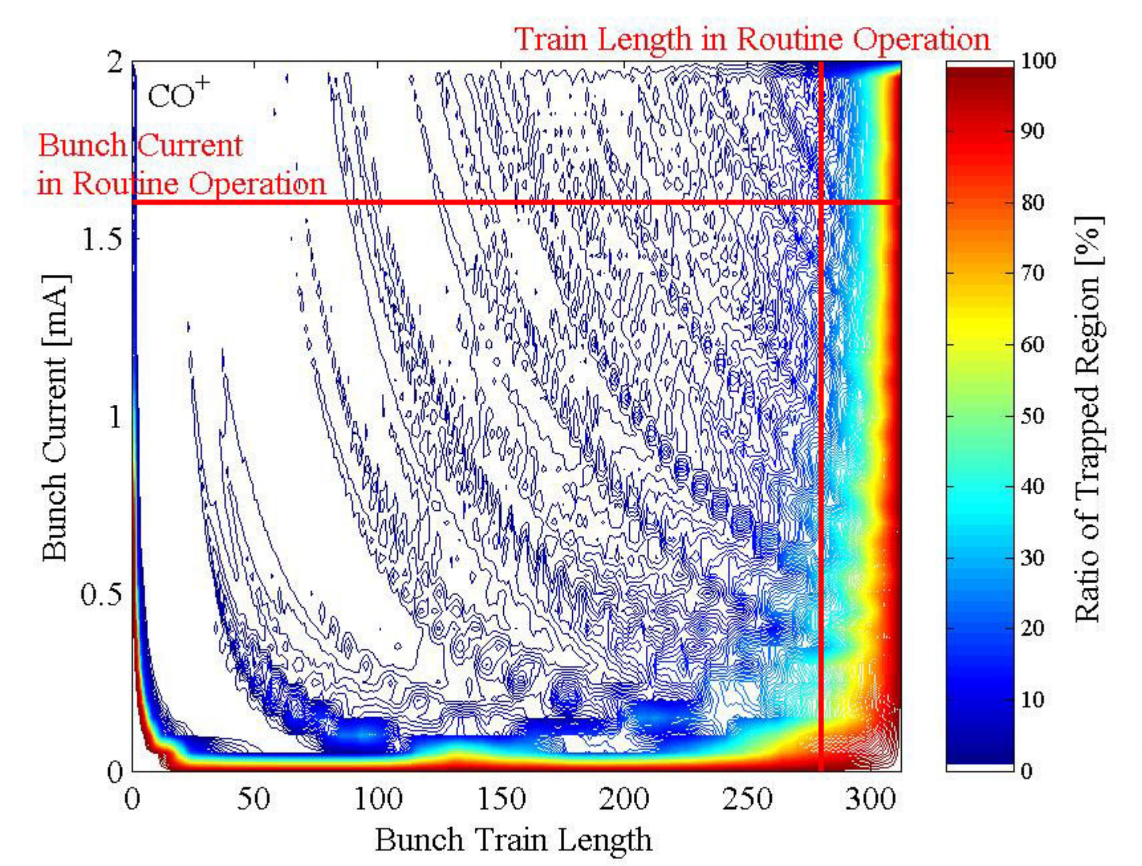
- The matrix M depends on the beam size.
- The beam size depends on the position in the ring.

We calculated the stability of the ions for the whole ring.

The calculation was performed in various configurations of the train.

(Train Length , Bunch Current)

We calculated a ratio of the total area of the regions where the ions are trapped to the area of the whole ring for various configurations of the train.



We divided the PF-ring circumference into pieces (10cm long)

… and estimated the stability of the ions in each piece.

Percentages of the regions where the ions are trapped in the ring.

The ion trapping phenomenon could be observed

in the multi-bunch condition in KEK-PF.

3.“Ion Twiss Parameters’’

Ions at a certain location in the storage ring are affected by a periodic focusing force with a period t_{rev} corresponding to the configuration of the bunch train.

Ion motion can be discussed with a method similar to that for the betatron oscillation in circular accelerators.

$\beta_i(\tau)$ … Ion betatron function

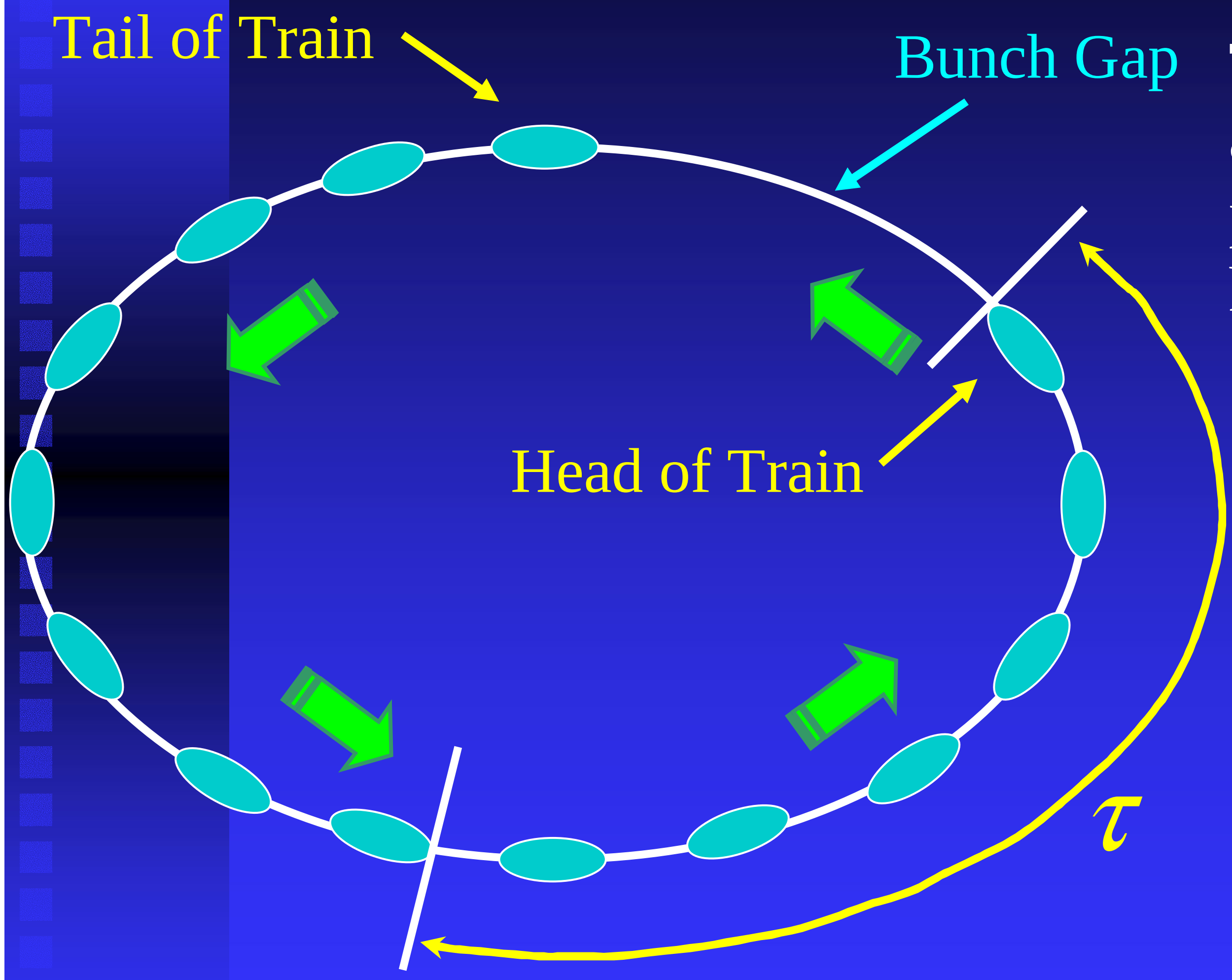
$$\beta_i(\tau) = \beta_i(\tau + t_{rev})$$

τ … Time lapse from the passage of a bunch head

The vertical displacement y_i of the ion at time t …

$$y_i(t) = y_0\sqrt{\beta_i(\tau)}\cos\phi(t)$$

Definition of Parameter



The parameter τ is defined as the time lapse from the passage of the bunch train head.

β_i …

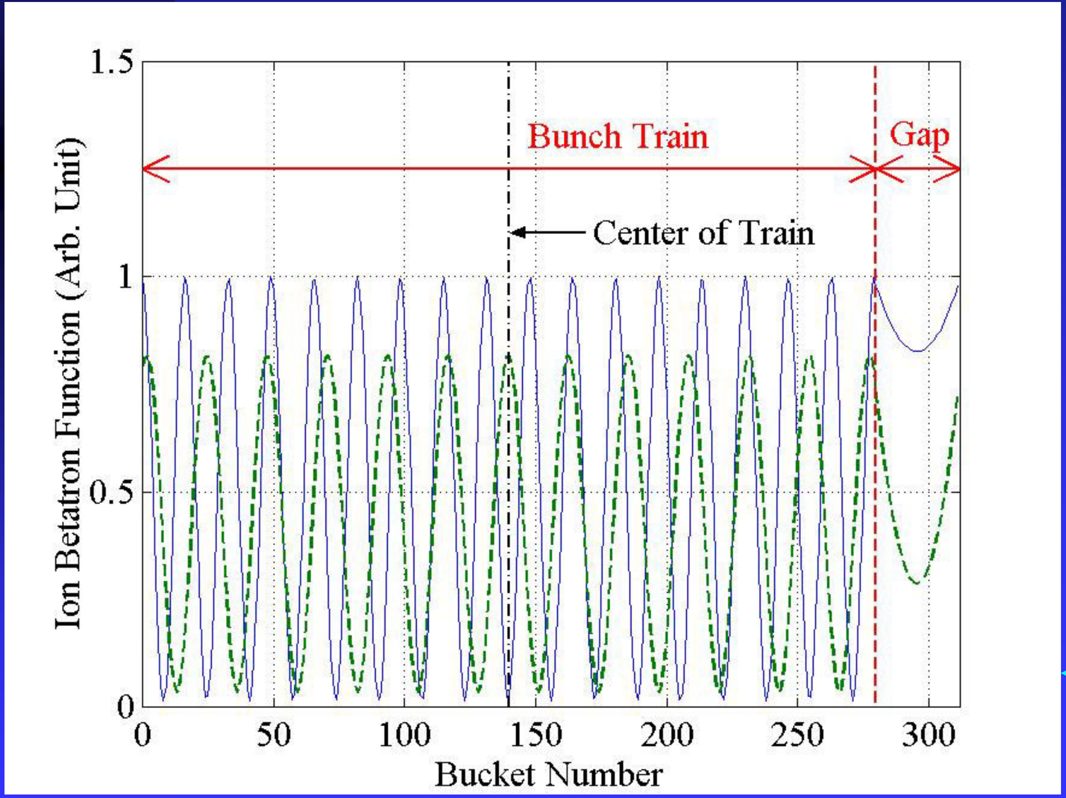
- is defined as
  a function of τ
- is a periodic function
  whose period is t_{rev}

Ion Betatron Function

280 Bunches + 32 Empty Buckets

I_{bunch} = 1.6 mA (I_{tot} = 450 mA)

CO^+ Ion



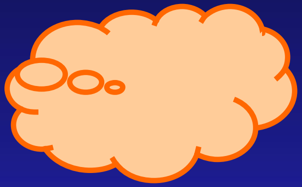
The bunch train has a mirror symmetry around the center of the train.

The betatron function of the ion is also mirror symmetric around the center of the train.

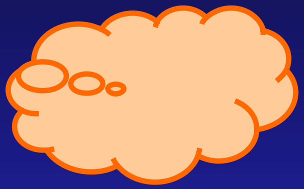
β_is at two different locations in the ring

What Does β_i Mean ?

$$\text{Oscillation amplitude of the ion} \propto \sqrt{\beta_i}$$



$$\text{Size of an ion cloud} \propto \sqrt{\beta_i}$$



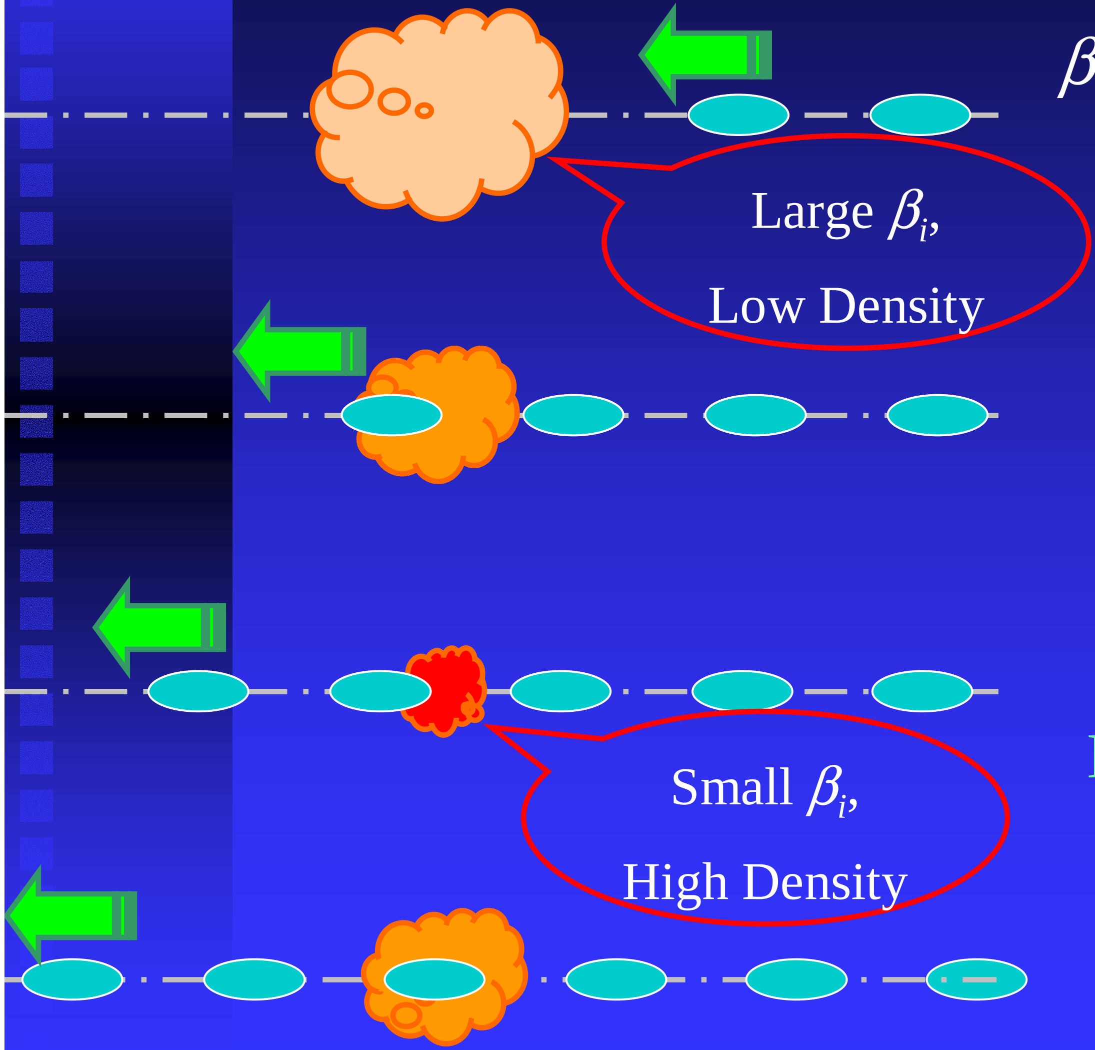
Meandering of β_i along the bunch train means

the size of the ion cloud changes during passage of the train.

….. Then, what occurs?

Modulation of trapped ion density along the bunch train could occur.

Modulation of Ion Density along the Bunch Train



β_i changes along the bunch train.

↓

Size of the ion cloud changes along the train.

↓

Modulation of the ion density occurs along the train.

4. Tune Shift Due to the Trapped Ions

We assume the horizontal/vertical size $\Sigma_{x,y}(\tau)$ of the ion-cloud

is written as : $\Sigma_{x,y}(\tau) = \underline{\varepsilon_{x,y}(\tau)}\sigma_{x,y}$ $\left(\sigma_x, \sigma_y\right)$... Beam Size

Modulation factor due to the oscillation of the trapped ions

Tune Shift Due to the Ion Trapping :

$$\Delta\nu_y(\tau) = \frac{r_e E_0}{2\pi E}\lambda_e \eta \int_C \frac{\beta_y(s)}{\Sigma_y(\tau)\left(\Sigma_x(\tau) + \Sigma_y(\tau)\right)} ds$$

λ_e ... Averaged line density of the electrons

η ... Neutralization factor

β_y ... Betatron function of the beam

Tune Shift that Depends on the Position in the Bunch Train

$$\Delta\nu_y(\tau) = \frac{r_e E_0}{2\pi E}\lambda_e \eta \int_C \frac{\beta_y(s)}{\varepsilon_y(\tau)\sigma_y\left(\varepsilon_x(\tau)\sigma_x + \varepsilon_y(\tau)\sigma_y\right)} ds$$

$$\approx \frac{\Delta\nu_y^0}{\varepsilon_y(\tau)}$$

………. We assume $\Sigma_x = \sigma_x$ $\left(\varepsilon_x(\tau) = 1\right)$

$$\Delta\nu_y^0 = \frac{r_e E_0}{2\pi E}\lambda_e \eta \int_C \frac{\beta_y(s)}{\sigma_y\left(\sigma_x + \sigma_y\right)} ds$$

….. Tune shift in the classical theory of ion trapping

Tune shift that depends on the position in the bunch train can be introduced with the modulation of the trapped ion density.

Modulation Factor $\varepsilon(\tau)$

Because the impulse acting on the ions depends on the beam size where the ions are trapped, β_i depends on location in the ring.

$$\text{The size of the ion cloud} \propto \sqrt{\beta_i(\tau)}$$

$$\text{The ratio } 1/\varepsilon_y(\tau) \propto \text{an average of } 1/\sqrt{\beta_i(\tau)}$$

over the whole ring

$$\frac{1}{\varepsilon_y(\tau)} = b_0 \int_C \frac{1}{\sqrt{\beta_i(\tau)}} ds, \qquad \text{..... Modulation of Ion Density}$$

$$\frac{1}{n_b t_{RF}} \int_0^{n_b t_{RF}} \frac{1}{\varepsilon_y(\tau)} d\tau = 1. \qquad \text{..... Normalization}$$

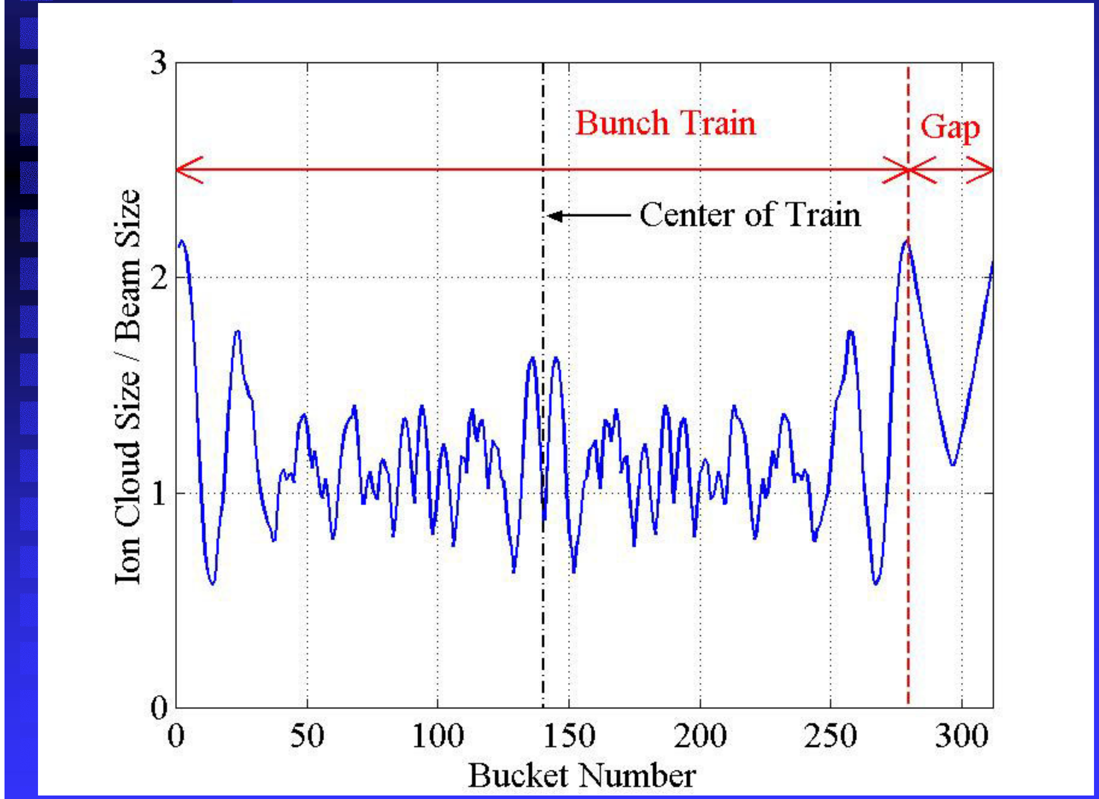
Change in Size of the Ion Cloud

280 Bunches + 32 Empty Buckets

I_{bunch} = 1.6 mA (I_{tot} = 450 mA)

CO^+ Ion

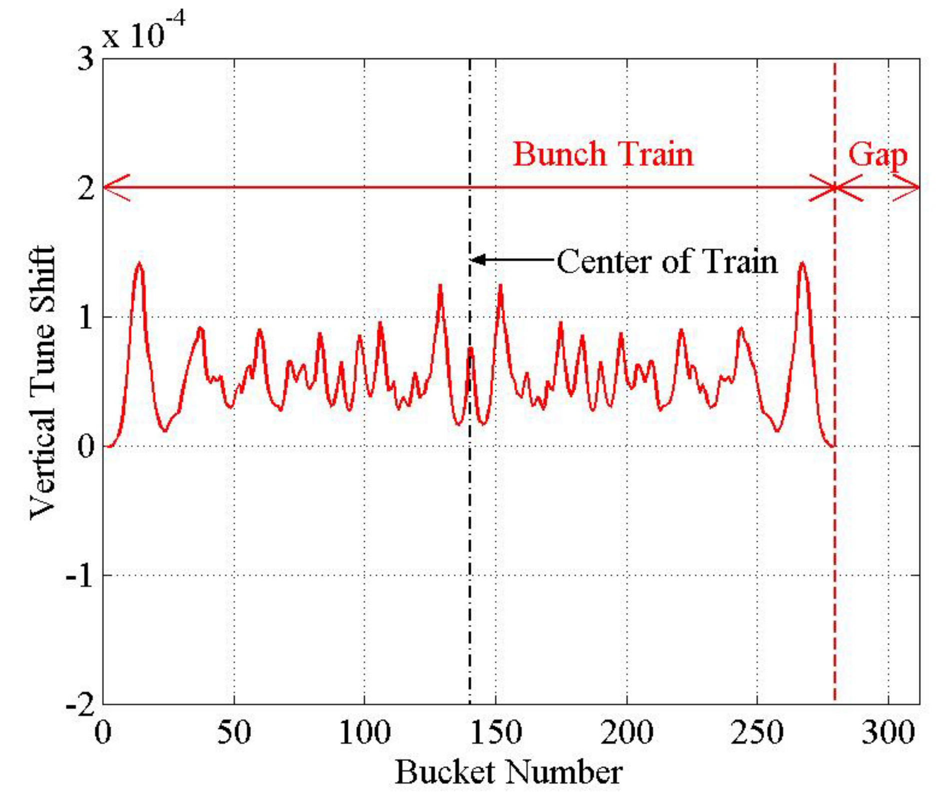
The multi-bunch condition in KEK-PF



It is supposed …

- the ion-cloud repeats expansions and shrinks along the bunch train.
- size of the ion cloud especially increases when the head/tail of the train passes.

Change in Vertical Tune along the Bunch Train



(Ordinate in this figure corresponds to the tune shifts from the tune of the first bunch.)

To obtain the change in the tunes along the bunch train from the modulation of the ion density,

Neutralization factor …

$\eta = 1.0 \times 10^{-5}$ is assumed.

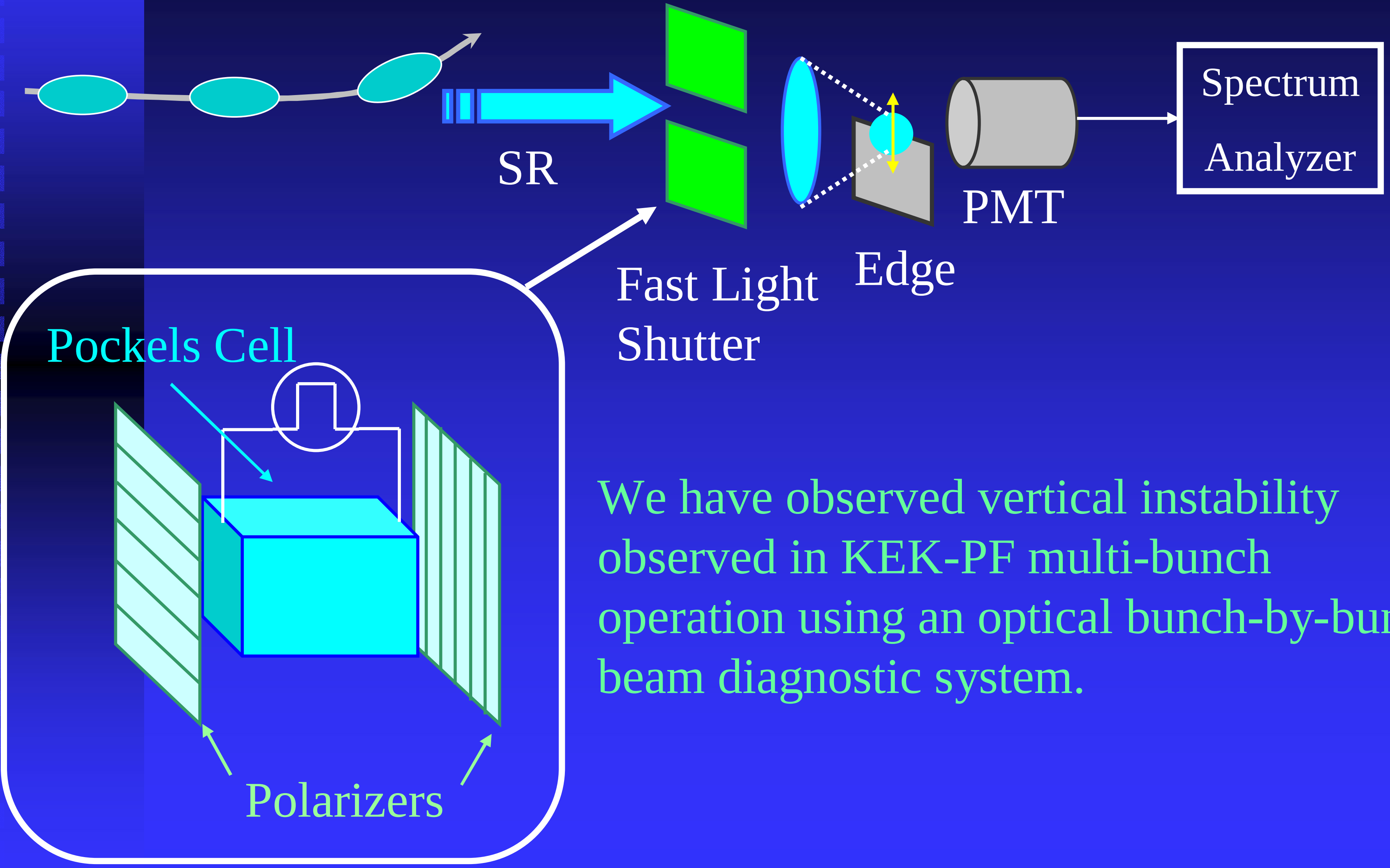
↑

Consistent value with an experimental result for measurement of η.

The tunes change along the train:

- rise in the head and fall in the tail

5. Bunch-by-Bunch Tune Measurement



We have observed vertical instability observed in KEK-PF multi-bunch operation using an optical bunch-by-bunch beam diagnostic system.

Experimental Results

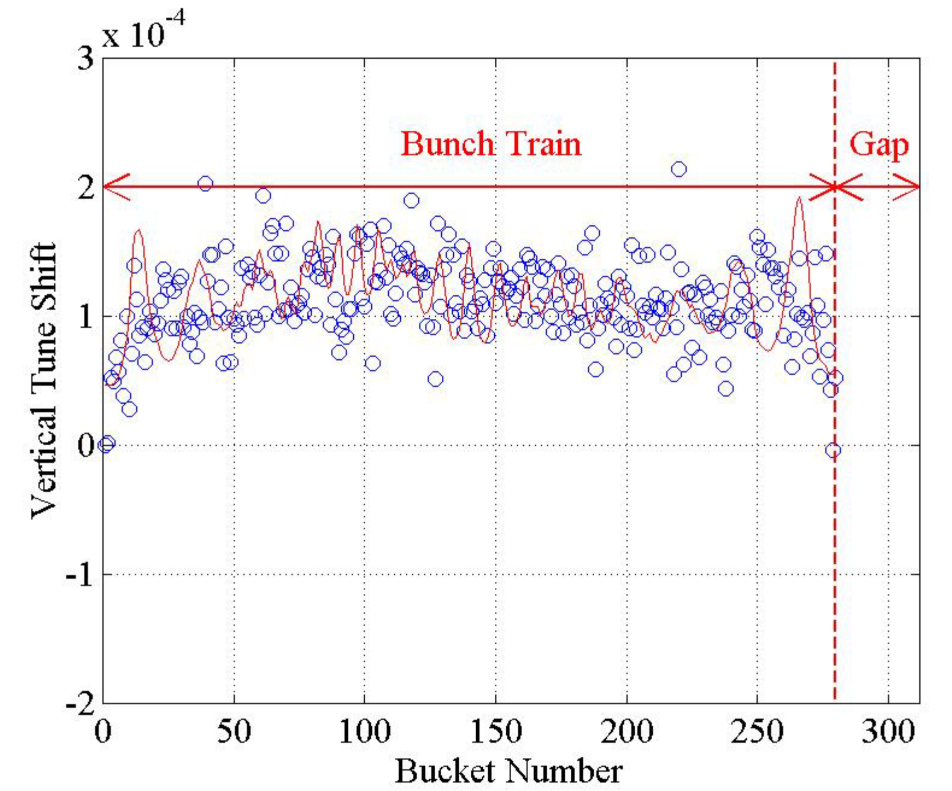
280 Bunches + 32 Empty Buckets

I_{bunch} = 1.6 mA (I_{tot} = 450 mA)

Ion species ... CO^+

$\eta = 9.4 \times 10^{-6}$ is assumed.

Change in the beam current during the experiment is considered in the theoretical calculation.



The change in the tunes along the bunch train shown in the experimental result reappears in the theoretical calculation.

Rise-up and fall-down of the tunes along the bunch train in the experiment also reappear in theoretical calculation.

Averaged Tune Shift along 20 bunches in the head

The. : $\Delta\nu = +5.1\times10^{-6}$ /Bunch

Exp.: $\Delta\nu = +4.0\times10^{-6}$ /Bunch

Averaged Tune Shift along 20 bunches in the tail

The. : $\Delta\nu = -6.4\times10^{-6}$ /Bunch

Exp.: $\Delta\nu = -2.1\times10^{-6}$ /Bunch

The experimental results agreed with the theoretical prediction.

Modulation of the ion density due to passage of the bunch train causes the periodic structure of the tunes along the train.

5. Summary and Next Step

- We tried a theoretical model that gives modulation of the ion density due to passage of the bunch train with the theory of betatron oscillation in circular accelerators.
- Experiments of bunch-by-bunch beam diagnostics in KEK-PF supported the theoretical prediction.

Next Step ...

Bunch-by-Bunch & Turn-by-Turn Beam Diagnostics in KEK-PF

Bunch-by-Bunch Beam Diagnostics in IMS-UVSOR