

# Longitudinal Wake due to Electron Cloud

*Giovanni Rumolo, Frank Zimmermann, CERN*

- Introduction
- Computational approach
- Results
- Conclusion

# Plasma Physics Estimate

maximum electric field in a plasma ('cold wavebreaking'):

$$E \approx \frac{m_e c \omega_p}{e}$$

where

$$\omega_p = \sqrt{\frac{4\pi \rho_e e^2}{m_e}}$$

in engineering units

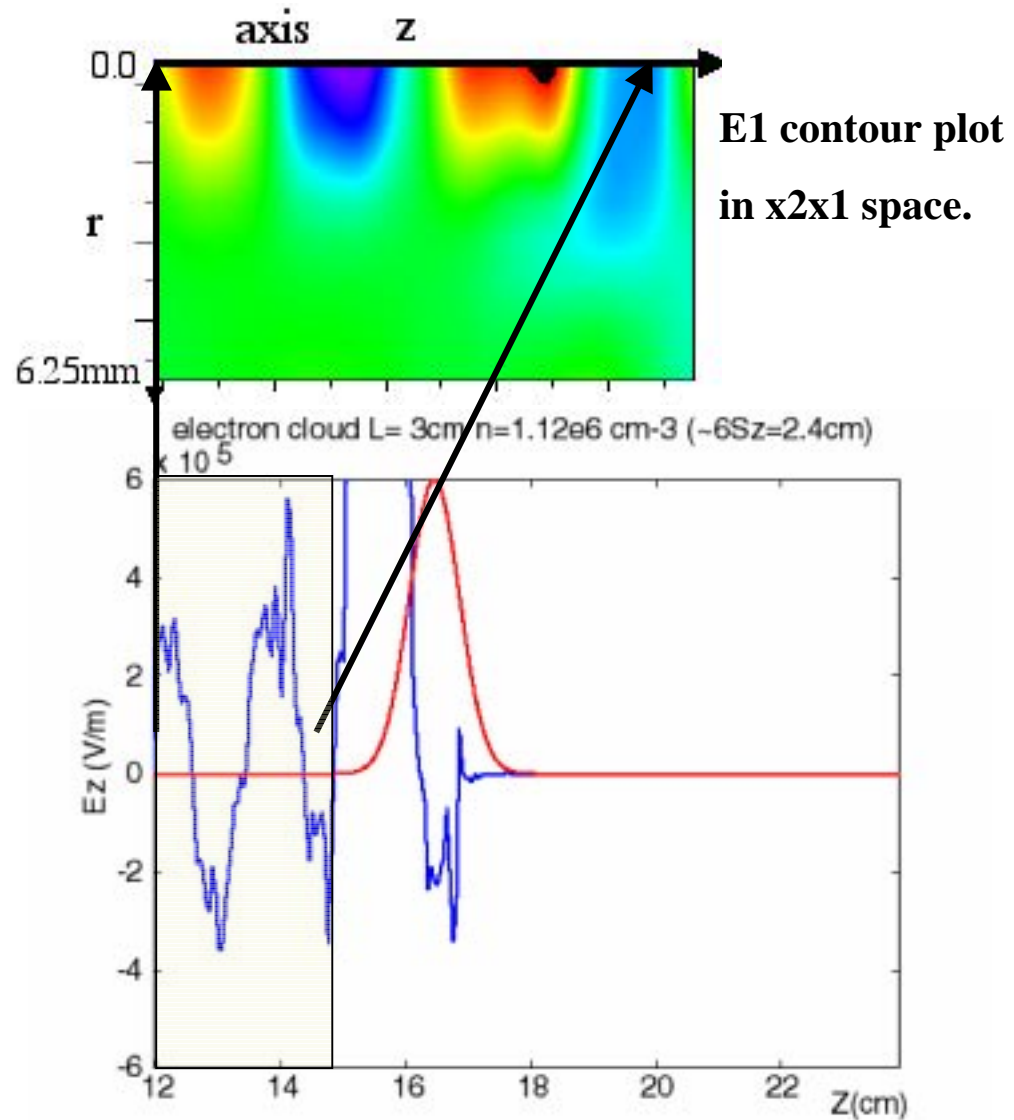
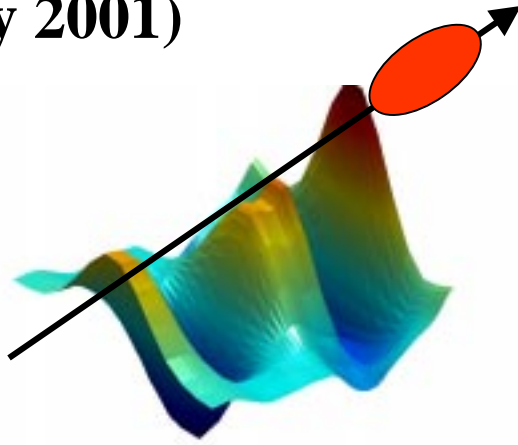
$$E \approx \sqrt{(\rho_e)} \text{ V/cm} \approx 100 \text{ kV/m}$$

expect enormous effect!?

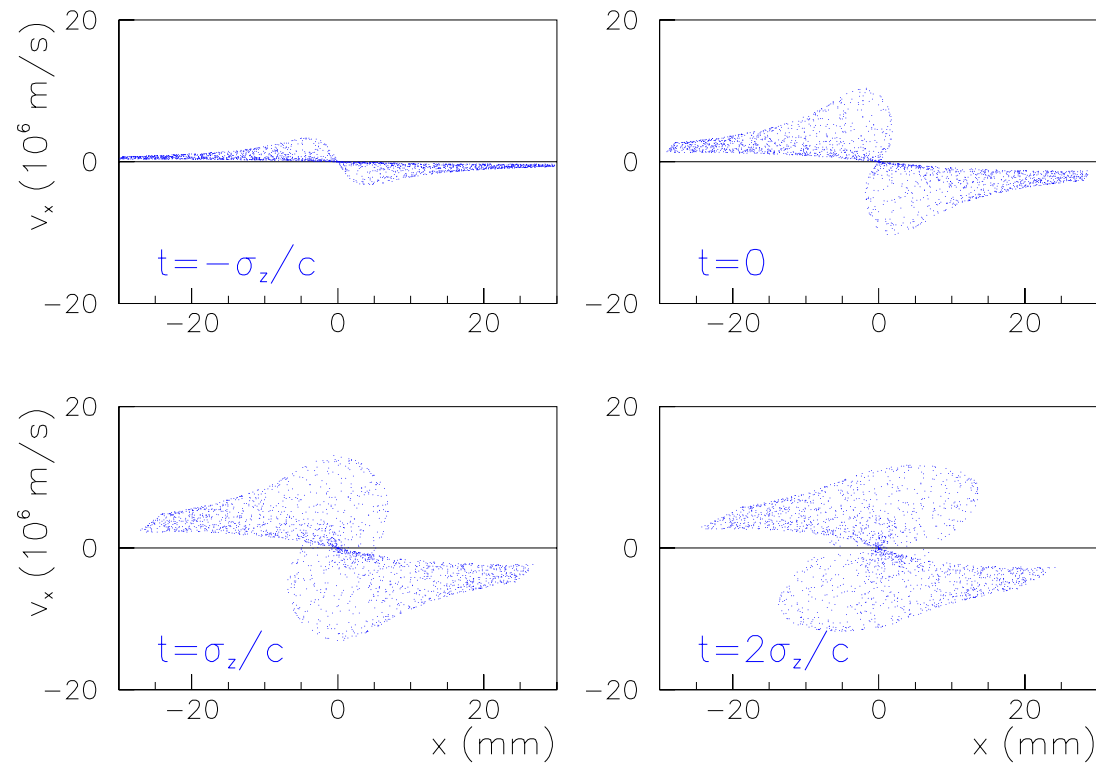
**PIC simulations**  
(S. Lee, T. Katsouleas, USC,  
May 2001)

longitudinal el. field  $E_1$  in V/m

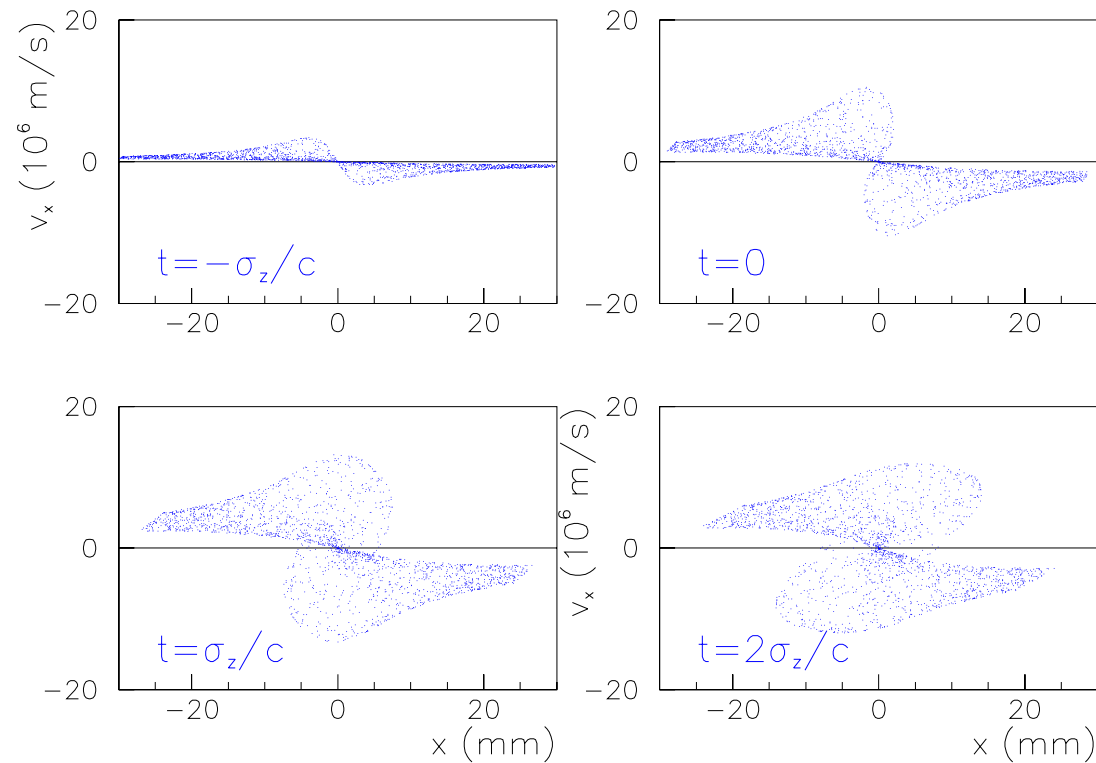
electron cloud density  $1.12 \times 10^6 \text{ cm}^{-3}$



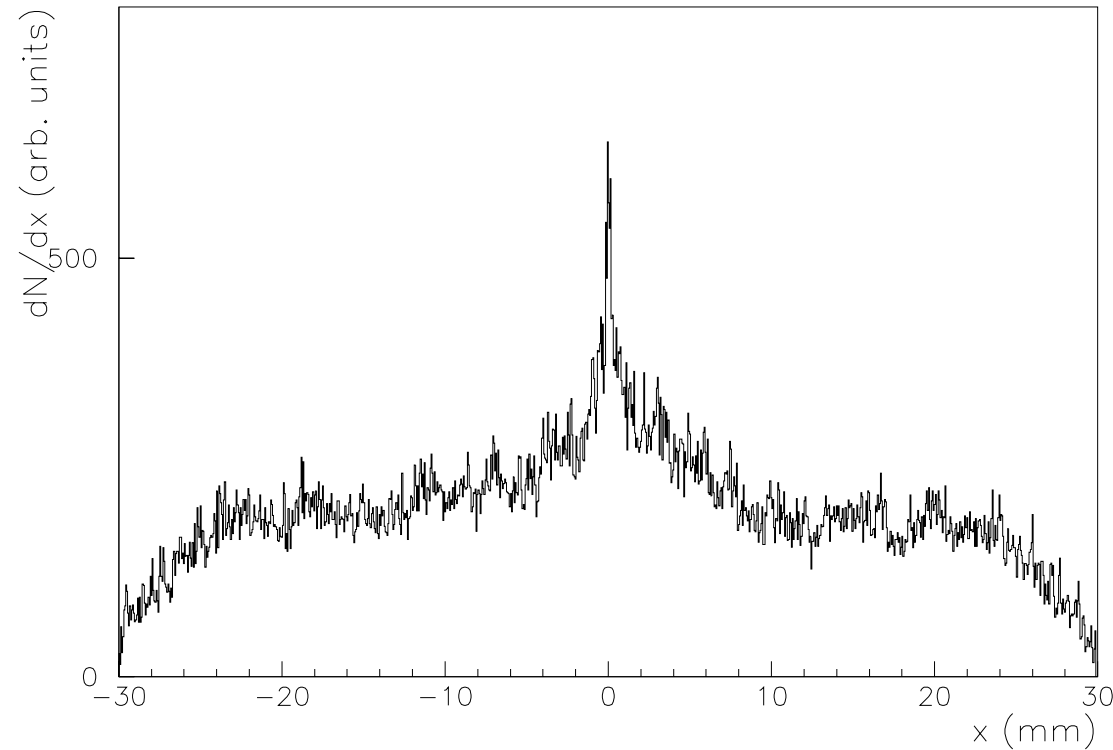
$R = 0.0 \text{ mm}$



Snap shots of horizontal electron phase space during the passage of an SPS proton bunch computed by 2-D PIC simulation ( $N_b = 10^{11}$ ,  $\sigma_{x,y} = 3$  mm,  $\sigma_z = 0.3$  m,  $\rho_e = 10^{12}$  m $^{-3}$ ). **Electron self-field is included.**

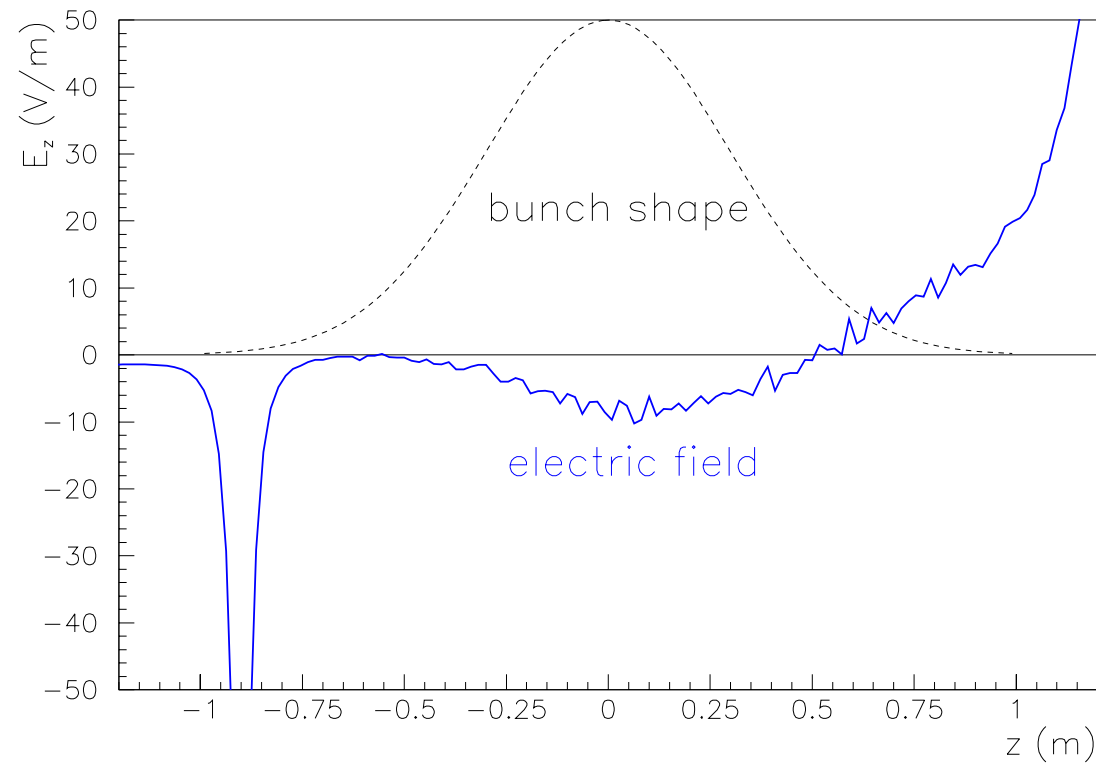


Snap shots of horizontal electron phase space during the passage of an SPS proton bunch computed by 2-D PIC simulation ( $N_b = 10^{11}$ ,  $\sigma_{x,y} = 3$  mm,  $\sigma_z = 0.3$  m,  $\rho_e = 10^{12}$  m $^{-3}$ ). **Electron self-field is *not* included.**



Horizontal electron distribution projected over  $\pm 2\sigma_z$  and  $\pm 10\sigma_y$  about the bunch center. Shown is a fraction  $1/250$  of the simulated macro-electrons.

- longitudinal electric field from 2-D PIC simulation
- $e^-$  concentrated at a single point
- bunch passes through
- different time steps correspond to different positions along the bunch
- compute field on 3-D grid, by identifying time with  $z$
- to get local field  $E_z$  apply reduction factor  $\Delta z/C$  to number of electrons
- assume a uniform  $e^-$  distribution in front of the bunch



Longitudinal electric field due to electron cloud for a Gaussian bunch in the SPS. Bunch head is on the left.

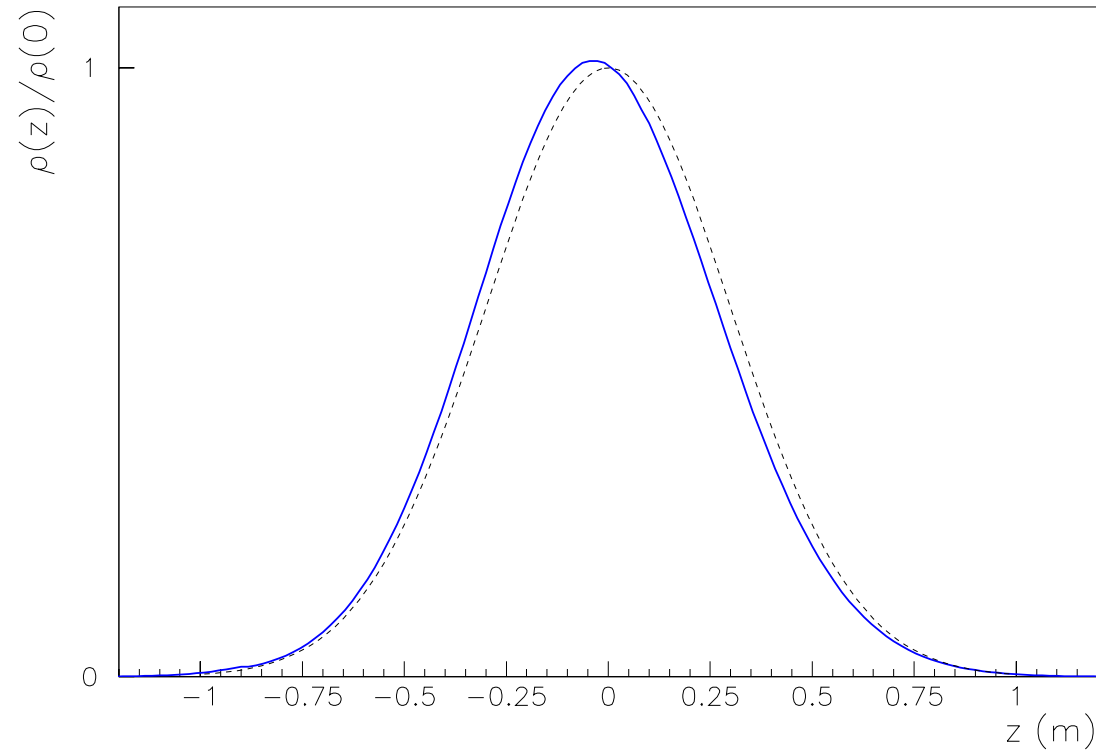


Compute bunch density expected from potential well as

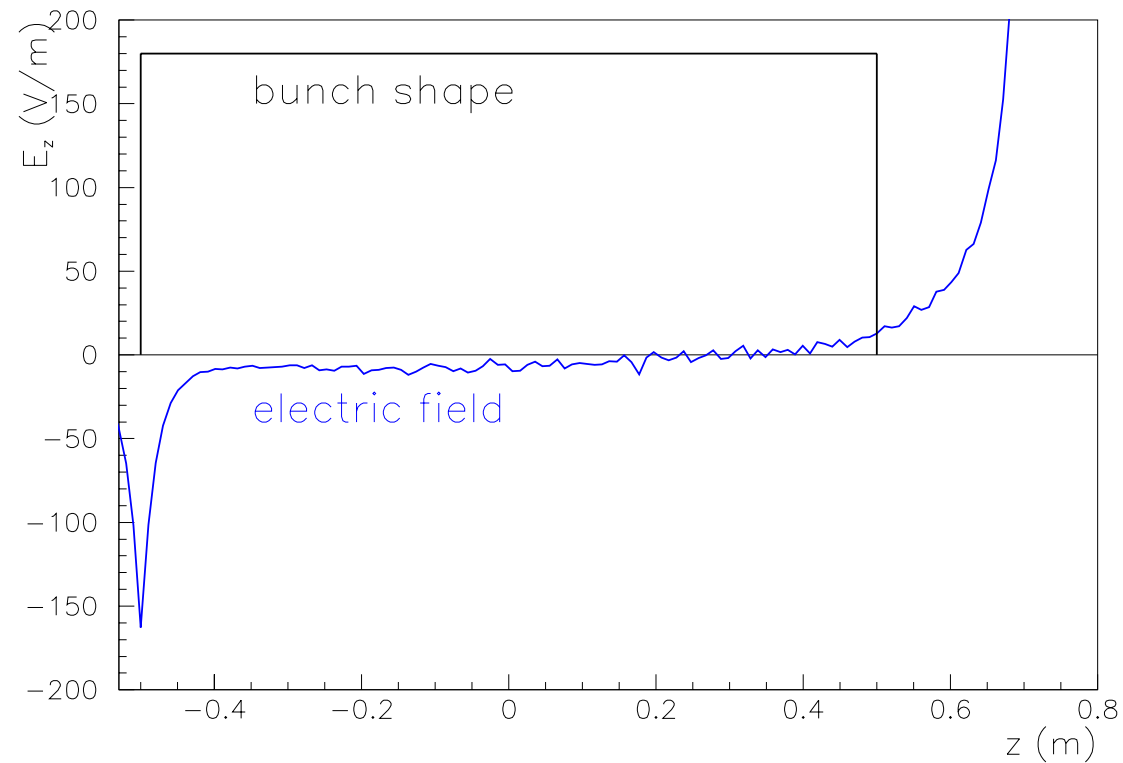
$$\begin{aligned}\rho(z) &= \rho(0) \exp \left[ -\frac{1}{2} \left( \frac{\omega_s z}{\eta c \sigma_\delta} \right)^2 \right. \\ &\quad \left. - \frac{r_0}{\eta \sigma_\delta^2 \gamma C} \int_0^z dz' \int_{-\infty}^{z''} dz'' W_0(z'' - z') \right] \\ &= \rho(0) \exp \left[ -\frac{1}{2} \left( \frac{\omega_s z}{\eta c \sigma_\delta} \right)^2 - \frac{r_0}{\eta \sigma_\delta^2 \gamma C} \int_0^z dz' W_z(z') \right]\end{aligned}$$

(‘quasi-Haissinski solution’) where

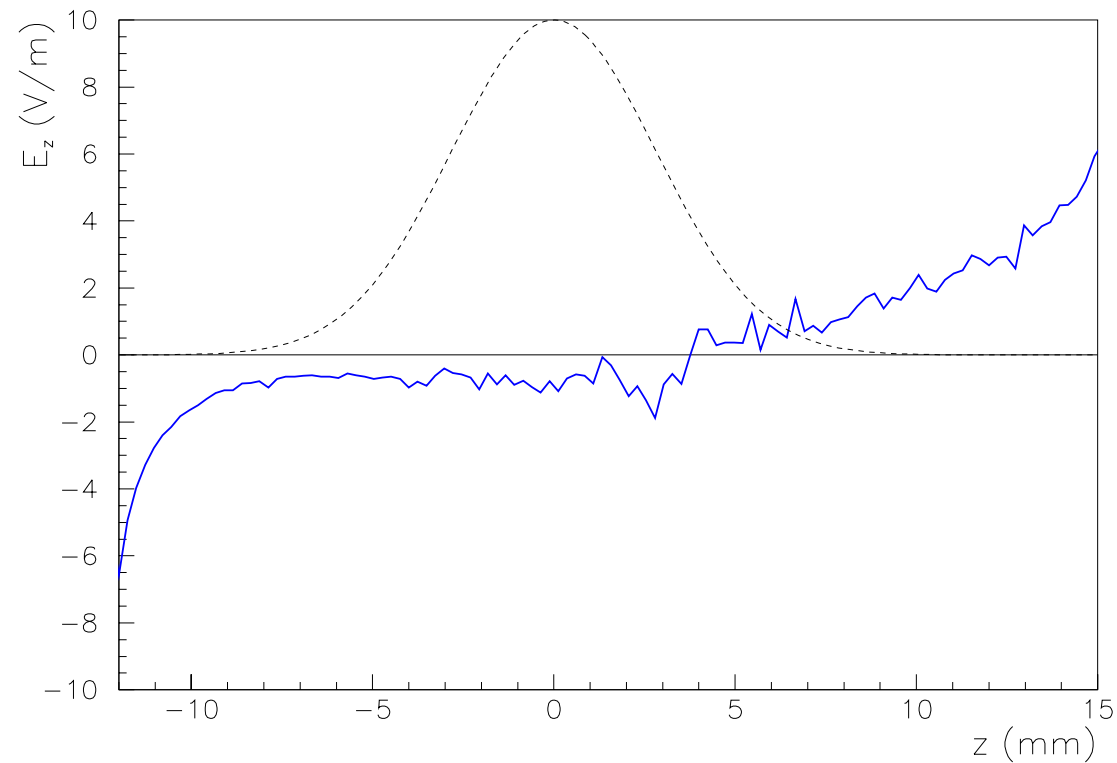
$$W(z) \approx -\frac{E_z(z)}{e} \left( \frac{4\pi}{Z_0 c} \right) C$$



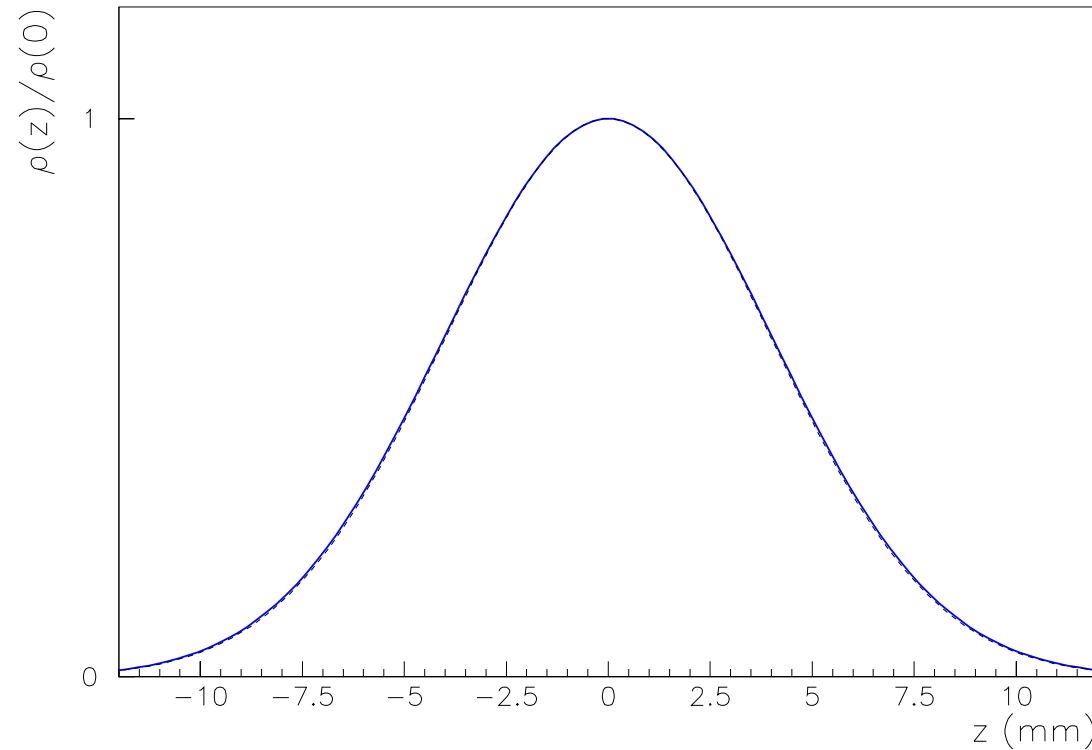
Equilibrium bunch density computed from the wake for a Gaussian bunch in the SPS. The Gaussian is slightly shifted.



Longitudinal electric field due to electron cloud for a flat bunch in the SPS. Bunch head is on the left.



Longitudinal electric field due to electron cloud for a Gaussian bunch in the KEKB LER. Bunch head is on the left.



Equilibrium bunch density computed from the wake for a flat bunch in the KEKB LER. The potential well distortion due to the cloud is insignificant.

## Conclusion

- longitudinal electric field due to  $e^-$  cloud computed using 2-D PIC code
- effect is smaller than expected, and negligible at KEKB