MODULATION OF TRAPPED ION DENSITY DUE TO PASSAGE OF BUNCH TRAIN IN KEK-PF ELECTRON STORAGE RING

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Abstract

An ion-related phenomenon that depends on a position in a bunch train is discussed. Density of trapped ions are modulated by periodic passage of a bunch train, and the modulation causes the phenomenon which has dependence on the position in the train. We tried a theoretical model that gives the modulation of the ion density caused by the passage of the train, and obtained theoretical value of change in vertical tunes along the train for multi-bunch operation in PF-ring. Bunch-by-bunch tune measurement supported the theoretical prediction.

1 INTRODUCTION

In the “classical” theory of ion trapping[1, 2], a stability condition of the ion motion and tune shift due to the trapped ions are discussed. In the theory, a size of an ion cloud is treated as constant and equal to a beam size; therefore, the theory does not treat a phenomenon in which the ion density varies along a bunch train.

The gas ions that are stably trapped by the beam, however, are affected by periodic force due to passage of the beam and oscillate around beam orbit. The oscillation causes change in size of an ion-cloud, and a modulation of the trapped ion density along the bunch train occurs. We have tried a theoretical model that gives the modulation of the ion density due to the passage of the train. In the model, the theory of betatron oscillation in circular accelerators was applied to the motion of the ions, and the change in the size of the ion-cloud along the train was derived from “betatron function of the ions”. Theoretical prediction of change in vertical tunes along the train due to the modulation of the ion density had good agreement with bunch-by-bunch tune measurement in the PF-ring.

2 ION MOTION

2.1 Equation of Motion

According to the classical theory[2], vertical displacement $y_i(t)$ of the ion at a position in a ring affected by a bunch can be expressed as

$$\begin{aligned}\begin{pmatrix} y_i(t+t_{RF}) \\ \dot{y}_i(t+t_{RF}) \end{pmatrix} &= \begin{pmatrix} 1 & t_{RF} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}\begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix} \\ &= DB\begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}, \end{aligned} \quad (1)$$

where t_{RF} is the bunch spacing and a denotes the impulse due to the passage of the bunch[2]. A dot in the equation represents a time derivative. In Eq.(1), the matrices B and D represent a thin-lens kick by the bunch and a drift between the bunches, respectively. The matrix for passage of the whole bunch train that consists of bunches with the same beam current can be written as the product of DB and D, namely,

$$\begin{aligned}\begin{pmatrix} y_i(t+t_{rev}) \\ \dot{y}_i(t+t_{rev}) \end{pmatrix} &= D^{h-n_b}\,(DB)^{n_b}\begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix} \\ &= M\begin{pmatrix} y_i(t) \\ \dot{y}_i(t) \end{pmatrix}, \end{aligned} \quad (2)$$

where t_{rev} is the revolution period of the beam, n_b is the number of bunches in the train and h is the harmonic number, respectively.

2.2 Stability Condition of Ions

A stability condition for the ions can be written with the trace of the matrix M as $\frac{1}{2}\left|\mathrm{T}rM\right| \leq 1$[2]. Because the trace depends not only on the configuration of the bunch train but also on the beam size, the stability condition differs at different positions in the ring under the same configuration of the train. We calculated the ratio of the total area of the regions where the ions are trapped to the area of the whole ring in various configurations of the train. Figure 1 shows with contour lines the ratio for various configurations of the train. In the calculation, an ion species of CO^+, which is the main component of the residual gas molecules in the PF-ring, is assumed. As seen in the figure, ions are rarely trapped when the train length is shorter than 100 bunches and the bunch current is equal to 1.6 mA/bunch, the bunch current in routine operation.

In the case that the ion-motion is stable, ions can oscillate around the beam. The oscillation can be discussed with a method similar to that for the betatron oscillation in circular accelerators. Namely, an “ion betatron function β_i” with the period t_{rev} can be defined as a function of the time lapse τ from the passage of a bunch head. However, there are some differences between the usual betatron function of the beam and that of the ion; namely, the betatron function of the ion exists under condition in which the ion is affected by periodic focusing force only, whereas the beam is governed by periodic focusing/defocusing forces. Moreover, the betatron function of the ion can be defined as a function of the lapse time τ, whereas the betatron function of the beam can be defined as a function of the position in the ring.

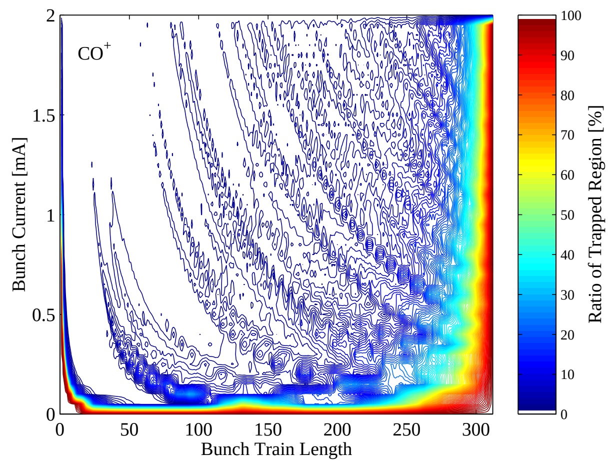

Figure 1: The ratio of the area of the regions where the ions are trapped to the area of the whole ring for various bunch train length (the total number of bunches).

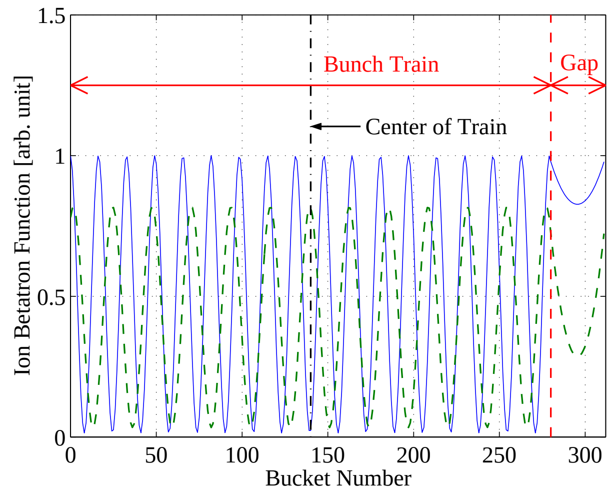

Figure 2: β_i at two different locations in the ring for 280 bunches.

2.3 Ion Betatron Function

The vertical displacement y_i of the ion at time t is given by

$$y_i(t) = y_0\sqrt{\beta_i(\tau)}\cos\phi(t), \tag{3}$$

where y_0 is a constant and $\phi(t)$ is the phase of the ion motion. Examples of β_i for multi-bunch mode (280 bunches + 32 empty buckets) with a bunch current of 1.6 mA/bunch are shown in Fig. 2. In the calculation of β_i, a CO^+ ion is assumed. Because the impulse due to the passage of a bunch depends on the beam size where the ions are trapped, a wiggle pattern of β_i depends on the position where the ions are trapped. Two different curves in the figure correspond to the β_i at two different locations in the ring. As seen in the figure, peaks and valleys in these two curves do not coincide with each other in the middle part of the train; however, the patterns of the β_i tend to decrease along the bunch train head and increase along the tail.

3 TUNE SHIFT

3.1 Change in Tune along the Bunch Train

Similar to the discussion for the beam in circular accelerators, a size of an ion cloud is proportional to $\sqrt{\beta_i(\tau)}$, and the size of the ion cloud varies along the bunch train because the β_i wiggles along the train as seen in the Fig. 2. Now we assume that the size of the ion cloud $\Sigma(\tau)$ is proportional to the beam size σ; namely,

$$\Sigma_{x,y}(\tau) = \epsilon_{x,y}(\tau)\sigma_{x,y}, \tag{4}$$

where $\epsilon_{x,y}(\tau)$ represents change in the size of the cloud along the train. If we assume that the horizontal motion of the ions is negligible ($\epsilon_x(\tau) = 1$), the tune shift along the bunch train is simply written as

$$\Delta\nu_y(\tau) = \frac{\Delta\nu_y^0}{\epsilon_y(\tau)}, \tag{5}$$

where $\Delta\nu_y^0$ is the tune shift in the classical theory of the ion trapping [2]:

$$\Delta\nu_y^0 = \frac{r_e E_0}{2\pi E}\lambda_e\eta\int_C \frac{\beta_y(s)}{\sigma_y(s)\left(\sigma_x(s)+\sigma_y(s)\right)}ds, \tag{6}$$

where r_e is the classical electron radius, E_0 the rest mass of the electron, E the total energy of electron, λ_e the averaged line density of electrons, η the neutralization factor, and β_y the betatron function of the beam, not the function of the ions, respectively. The integral in Eq. (6) is taken over the area C where the ions are stably trapped around the beam. The ratio $1/\epsilon_y(\tau)$ in Eq. (5) represents a modulation of the trapped ion density along the train. Because the size of the ion cloud is proportional to $\sqrt{\beta_i}$, the ion density is proportional to an average of $1/\sqrt{\beta_i}$ over the whole ring, namely,

$$\frac{1}{\epsilon_y(\tau)} = \frac{\int_C \frac{1}{\sqrt{\beta_i(\tau)}}ds}{\frac{1}{n_b t_{RF}}\int\left(\int_C \frac{1}{\sqrt{\beta_i(\tau)}}ds\right)d\tau}. \tag{7}$$

Equation (5) is consistent with the classical theory because an averaged value of the tune shift over the whole bunch train equals $\Delta\nu_y^0$; namely, Eq. (7) satisfies $\frac{1}{n_b t_{RF}}\int\frac{1}{\epsilon_y(\tau)}d\tau = 1$.

3.2 Ion Cloud Motion

As given by Eq. (4), the factor $\epsilon_y(\tau)$ represents the change in the size of the ion cloud along the train. Figure 3 shows the change in the size of the cloud in the same configuration of the bunch train in Fig. 2. Similar to the $\beta_i(\tau)$, the change in the size of the ion cloud along the train has mirror symmetry around the center of the train and has a period of t_{rev}. The cloud has a tendency to expand and shrink repeatedly during the passage of the middle part of the train, and its size tends to become remarkably large (2 times as large as the beam size) in both the head and the tail of the train. The cloud rapidly shrinks and expands after the passage of the head and before the passage of the tail, and that indicates the effect due to the change in the ion density can be observed in both the head and the tail of the train experimentally.

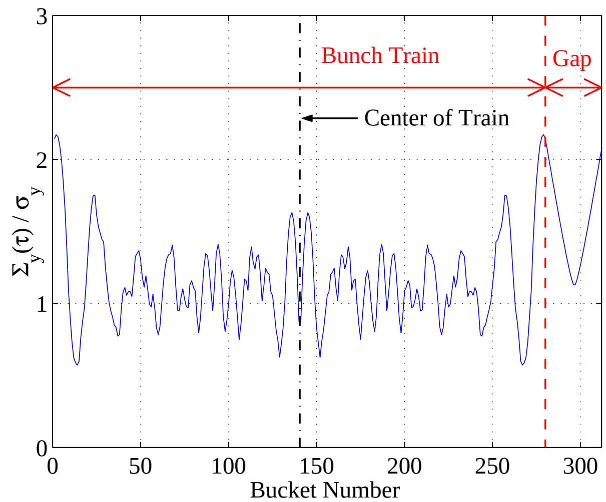

Figure 3: The change in the size of the ion cloud along the bunch train. The ordinate corresponds to the factor $\epsilon_y(\tau)$.

3.3 *Bunch-by-Bunch Tune Measurement*

We have observed a vertical instability in the PF-ring multi-bunch mode (280 bunches) with an optical bunch-by-bunch beam diagnostic system [4] and analyzed the vertical tunes of individual bunches. The experimental result and the theoretical value are shown in Fig. 4. The figure shows tune shifts from the tune of the first bunch as a function of the bunch position in the train. In the calculation of the theoretical values, an ion species of CO^+ and a neutralization factor of 9.4×10^{-6}, which is consistent with an experimental result for measurement of the neutralization factor in the PF-ring. In the calculated tune shift in the figure, an asymmetry around the center of the bunch train can be seen although the β_i is symmetric, as seen in Fig. 2. This is because the asymmetry arises from the change in the beam current during the measurement.

We calculated the averaged tune shifts along 20 bunches in the head. A theoretical value of $\left(\frac{\Delta\nu_y}{\Delta n}\right) = 5.1 \times 10^{-6}$ was obtained, which agreed closely with the experimental value of $\left(\frac{\Delta\nu_y}{\Delta n}\right) = 4.0 \times 10^{-6}$. On the other hand, the averaged tune shifts along 20 bunches in the tail was -6.4×10^{-6} from the theory and -2.1×10^{-6} from the experiment. The agreement is not so good; however, a tendency to decrease the tunes in the tail of the train agrees with each other.

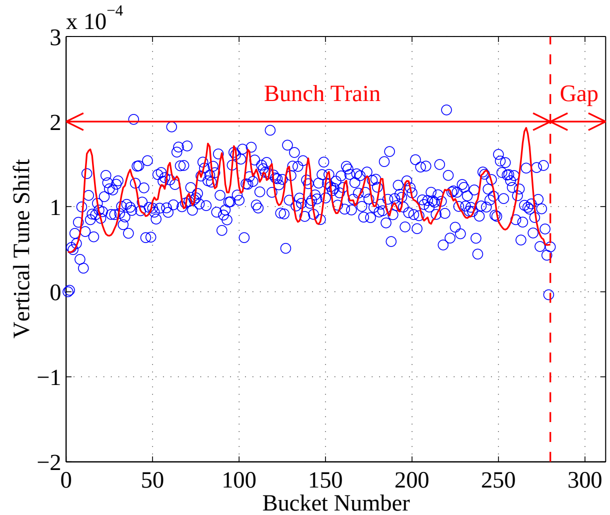

Figure 4: The change in the vertical tunes along the 280 bunch train. Circles and a curve correspond to the experimental results and the theoretical values.

4 SUMMARY

We have tried a model in which the modulation of the ion density during the passage of the bunch train is taken into account. The model treats change in the size of the ion cloud due to the passage of the train. The theoretical value of the change in the vertical tunes along the train agrees with the experimental result in which a vertical instability has been observed with an optical bunch-by-bunch beam diagnostic system in the PF-ring.

5 ACKNOWLEDGMENTS

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